Estimating Sector Concentration Risk from Practitioners' View[*]

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Abstract

For the overall model performance, we found that, under 'infection model' by Duellmann (2006), it can estimate the unexpected loss under the multi-sector Monte Carlo simulation (true model) fairly well with Thai historical data between 2003 and 2009. On the other hand, 'diversification-adjustment' approach can estimate the unexpected loss under true model relatively poor with Thai historical data. On the other hand, in terms of the model performance by each sector, the infection model, Moody's Binomial Expansion Technique, Asymptotic Risk Factor model, and 'diversification-adjustment' approach perform from best to worst.

1. Introduction

Since 2008, Bank of Thailand has required all financial institutions in Thailand to implement Basel II under either standardized approach, SA, (the basic level) or internal-rating based approach, IRB, (the more sophisticated one). In general, as the pattern of Basel II adoption internationally, the big banks will early adopt IRB approaches following subsequently with the smaller ones.

Unfortunately, unlike SA approach, in which there is a level playing field for all size of financial institutions due to its simplicity, IRB approach, particularly Pillar II, the bigger banks generally has an advantage over the smaller ones. The reason is that many required IRB risk management tools ranging from concentration risk, economic capital modeling, toward advanced stress testing need large amount of investment to build both IT and risk management infrastructure to perform sophisticated analysis on the regular basis.

Since the customization of the commercial Basel II software system customized catering for smaller banks feasibly to purchase is nearly impossible, the investment for the small banks for full scope of IRB does not necessarily make a positive return in terms of profitability. This paper aims to make this gap become narrower, especially in the area of concentration risk

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particularly focusing on the sector concentration. At present, under SA Basel II, Bank of Thailand has used 'the limit' as the tool to manage the sector concentration risk in the banking system. However, under the IRB approach, Pillar II recommends the method that is more sophisticated and more data-intensive to manage concentration risk.

This paper adapts the so-called infection model, according to Duellmann (2006), to estimate capital ratio for financial institutions to set aside this amount as cushion on unexpected loss. This approach would probably be more suitable for smaller banks since this way of sector concentration estimation requires less amount of data and generally less resource than other available methods do. As the benchmark, the analysis based on the multi-factor 'factor adjustment' for the treatment of diversification of credit capital rules based on Cespedes et al (2005), Asymptotic Risk Factor model (ASRF) formula from Basel II, and Moody's Binomial Expansion Technique (BET), is also performed to evaluate their model performance.

The paper is divided into five sections. Starting with the introduction part, the second section makes the review on related literature in concentration risk area. The third section copes with the methodology, dividing into four sub-sections: multi-sector Monte Carlo or 'true model', infection model, Asymptotic Single Risk Factor or ASRF framework, and a model for the diversification adjustment according to Cespedes et al (2005). Then, the result with discussion is in the fourth section. Finally, the conclusion with Appendix illustrating default frequency diagrams of the infection model is in the final section.

2. Review Literature

In the banking industry, based on Basel II capital formula from Basel Committee on Banking Supervision (2005) or 'Asymptotic Single Risk Factor' (ASRF) formula, in which all financial institutions implementing Basel II are currently applied to assess their degree of risk, it will be able to accurately approximate the optimal amount of capital for their corresponding degree of risk, only if the assumption of 'Asymptotic Single Risk Factor' (ASRF) is valid. In other words, with the two conditions that no any single account is relatively large in comparison to others and the only single risk factor, which is the correlation between the return of the portfolio analyzed and its economic condition, are presented in the portfolio, the ASRF formula, based on the Vasicek formula according to Vasicek (1987), could estimate their portfolio risk level with a fairly accurate precision.

Unfortunately, in the real world, these two conditions are not easily met. Suppose that if the first one is violated due to concentration risk from too low granularity of loan accounts within their portfolio, the 'name concentration' method to capture the granularity of all customers in the whole portfolio.

For the second condition, violations of the “single systematic factor” assumption may be more difficult to discern, and also more difficult to address than imperfect granularity. In general, different industries can experience different cycles. These realities suggest that distinct industries ought to be represented by distinct (though possibly correlated) systematic risk factors. In this case, a particular bank may be heavily concentrated in its exposure to some of these risk factors and lightly concentrated to others. The extent to which a single-factor model (and, by extension, the IRB risk weights) understates economic capital depends on both the degree to which the bank is unbalanced in its industry exposures and the extent to which industry risk factors are correlated with one another. This form of credit concentration risk is known as sector concentration.

There is a growing body of literature that deals with the question of measuring the role of sector concentration on credit risk assessment, either explicitly or implicitly through the analysis of multi-factor portfolio models. In other words, there are two approaches to deal with sector concentration. The first approach comes from the realisation that risk is inherently multi-dimensional and focuses on developing multi-factor models. The thrust of this approach is to find ways to overcome the reliance of the models on Monte-Carlo simulations that are portfolio specific and not easy to generalise and to validate. Recent example includes Duellmann and Masschelein (2006).

The starting point of the second approach for dealing with sector concentration is that the gap between the economic capital assessed through a multi-factor model and a more parsimonious framework is of second-order importance and can be bridged by adjustments to the economic capital figure obtained in closed-form for the simpler model using readily available inputs. Examples include: the binomial expansion technique, the infection model, and the diversity score model. The approaches pioneered by Duellmann (2006) adapting on the work from Davis and Lo (1999), and Cespedes et al (2005) are applied to this line of research.

3. Methodology

The methodology section is divided into four parts: Multi-factor Capital, ‘Infection Model’, Asymptotic Single Risk Factor Model, and ‘Capital Diversification Approach’.

3.1 Multi-sector Monte Carlo General Framework

According to Duellmann and Masschelein (2006), the estimation of risk degree, henceforth the amount of capital, for financial institution is through Multi-Factor Monte Carlo simulation, based on the Merton-type credit model. This is under the viewpoint of corporate sustainability under the premise that the value of liability cannot exceed that of asset. The idea is to utilize the data in the sector level applying to Monte Carlo simulation with the multi-factor credit risk model.

In terms of terminology, let M denote the number of borrowers or loans in the portfolio, M_s the number of borrowers in sector s, where s the number of sectors and w_{si} the weight of the exposure of borrower i in sector s relative to the total portfolio exposure.

According to the multi-factor credit model, the unobservable, normalized asset return X_{si} of of the i-th borrower in sector s triggers the default event if it crosses the default barrier barrier γ_{si} . In other words, credit risk occurs only as a default event at the end of a one-year horizon, which is consistent with traditional book-value accounting. The corresponding unconditional default probability p_{si} is defined as

$$p_{si} = \mathrm{P}(X_{si} \leq \gamma_{si}) \qquad (1)$$

where X_{si} consists of two parts: the systematic part and the idiosyncratic part according to

$$X_{si} = r_s \, Y_s + (1 - r_s^2)^{0.5} \, \varepsilon_{si} \qquad (2)$$

where $s \in \{1, \ldots, S\}$ and $i \in \{1, \ldots, M_s\}$.

The first part is estimated by the systematic sector risk Y_s where can be expressed as a linear combination of independent, standard normally distributed factors $Z_1 , \ldots, Z_s$

$$\mathrm{Y}_s = \sum_{t=1}^{S} \alpha_{st} Z_t \quad \text{with} \quad \sum_{t=1}^{S} \alpha_{st}^2 = 1 \text{ for s} \in \{1, \ldots, \mathrm{S}\} \qquad (3)$$

The matrix $(\alpha_{st})_{1 \leq s,\, t \leq S}$ is obtained from a cholesky decomposition of the factor correlation matrix.

The second or idiosyncratic part is represented by an idiosyncratic risk factor ε_{si}, where ε_{si} follows a standard normal distribution. For the weight to be allocated between systematic and idiosyncratic part is assigned by the value of r_s, represented by the intra-sector asset correlation for each pair of borrowers.

The asset correlation ω_{st} for each pair of borrowers in sector s and t, respectively, can be shown to be given by

$$\omega_{st} = r_s \, r_t \, \rho_{st} = r_s \, r_t \sum_{n=1}^{S} \alpha_{sn} \alpha_{tn} \qquad (4)$$

In this paper, the cholesky decomposition is performed by using net profit data for industry sectors in Thailand using the quarterly Thai data from 2003 to 2009.

If a firm defaults, the amount of loss depends on the stochastic loss severity ψ_{si} whose realization is assumed to be known at the time of default. The credit losses of the whole portfolio are given by

$$L = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{st} \psi_{si} 1_{\{x_{si} \leq N^{-1}(p_{si})\}} \tag{5}$$

where $1_{\{.\}}$ gives the indicator function.

We assume the same expected loss severity $\mu = E[\psi_{si}]$ for all borrowers and that all idiosyncratic risk in loss severities is diversified away in the portfolio.

In summary, the model needs the following input parameters:

- relative exposure size w_{si} and default probabilities p_{si} of the i-th borrower in sector s
- the factor correlation matrix and
- the sector-dependent factor weight r_s

3.2 Infection Model

There are two steps in implementing the so-called 'Infection Model' with the first one to determine the value of parameters used in the simulation of the 'infection model' in the second step. Then, the second step is the process of 'infection model' simulation, in which the loss behavior from the model is compared with that from actual loan loss.

First step: it is to map from the actual distribution of default in loan portfolio into their approximated distribution. In other words, the profile of actual loan portfolio in financial institution should be mapped into that of hypothetical loan portfolio estimated by binomial distribution. In order to accurately project the latter distribution to capture the former distribution in a systemic way, this requires the estimation of the parameters determining the characteristics of the approximated distribution to model the actual loan characteristic.

For binomial expansion, it is the discrete probability distribution of the number of successes in a sequence of n independent default/non-default experiments, each of which yields default outcome with probability p. Such a default/non-default experiment is in general the basis for many types of credit risk modeling.

The parameters determining the characteristics of their binomial distribution is composed of two numbers*: average probability of default* and *modified diversity score* (the number of customer account adjusted for the correlation effect.)

The first parameter or *average probability of default* is estimated under the assumption that the total or average amount of loss in loan portfolio (first moment) is the same under both actual loan portfolio and the hypothetical loan portfolio.

For the detailed estimation, it works as follows. Let $A_{i,k}$ denote exposure k in sector i of the real portfolio, m the number of sectors, $n(i)$ the number of exposures in sector i, and D the modified diversity score.

For simplicity, it is assumed that every exposure belongs to a different borrower, eliminating the need to differentiate between exposures and borrowers. Let A refer to the total exposure, which is the same for both portfolios:

$$A = \sum_{i=1}^{m}\sum_{j=1}^{n(i)} A_{i,j}$$

Under the actual portfolio, its total loss is the sum of loss from each individual customer loss, in which it equals the product of his probability of default or P_i, his exposure at default or $A_{i,j}$, and his loss give default or *LGD*. At the same time, under the hypothetical portfolio, each individual customer loss is the product of correlation-adjusted number of customers or diversity score or D, the average probability of default or $\bar{p}$, the uniform exposure or A/D, and the loss give default or *LGD*. Both numbers has to be equal to each other:

$$\sum_{i=1}^{m}\sum_{j=1}^{n(i)} p_i \cdot A_{i,j} \cdot LGD = D \cdot \bar{p} \cdot \frac{A}{D} \cdot LGD \qquad \textbf{(6)}$$

From (6), it follows that $\bar{p}$ equals the weighted average PD of the real portfolio:

$$\bar{p} = \left(\sum_{i=1}^{m} p_i \sum_{j=1}^{n(i)} A_{i,j} \cdot \right) \cdot A^{-1} \qquad \textbf{(7)}$$

For the second parameter or *modified diversity score,* it is estimated under the assumption that the variance of amount of loss in loan portfolio (first moment) is the same under both actual loan portfolio and the hypothetical loan portfolio.

For the detailed estimation, it works as follows. Let $U_{i,j}$ denote the indicator function that signals a default of exposure j in sector i in the original portfolio and U_k a default of the k-th exposure in the hypothetical portfolio. Matching the variances of losses in the real and the hypothetical portfolio provides an explicit expression for the modified diversity score:

$$Var\left(\sum_{i=1}^{m}\sum_{j=1}^{n(i)} A_{i,j} \cdot LGD \cdot U_{i,j} \right) = Var\left(\frac{A}{D} \cdot LGD \cdot \sum_{k=1}^{D} U_k \right) \qquad \textbf{(8)}$$

Note that the value of LGD parameter does not affect the result of the moment matching because this parameter cancels out in (8) and (6).

Unlike the average of loss, in which the computation does not take the default correlation between customers into consideration, the variance of loss requires this input. In doing this, we define the pair-wise default correlation $\omega_{j,l}^{i,k}$ of two exposures k and l in sector i and j is the same within each sector (ω_i^{intra} , ω_j^{intra}) and between any two sectors (ω^{inter}):

$$\text{For } i \in \{1, \dots, m\}: \qquad \omega_{j,l}^{i,k} \begin{array}{ll} = 1 & : \ i = j \text{ and } k = 1 \\ = \omega_i^{intra} & : \ i = j \text{ and } k \neq 1 \\ = \omega^{inter} & : \ i \neq j \end{array} \qquad \textbf{(9)}$$

Let p_i denote the default probability, which is the same for all borrowers in sector i. Then from (8) it follows for the modified diversity score D that

$$D = \frac{A^2 \bar{P}(1-\bar{P})}{\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{n(i)}\sum_{l=1}^{n(j)} A_{i,k} A_{j,l} \omega_{j,i}^{i,k} \sqrt{p_i(1-p_i)p_j(1-p_j)}} \qquad \textbf{(10)}$$

These two inputs are all we need for Binary Expansion. However, for Infection Model, in addition to these two parameters on Binomial Expansion, the infectious parameter is also needed to be determined.

In determining the value of this infectious parameter, there are two steps needed to be taken:

First, the Value-at-Risk (VaR) or 99.9% quantile of the loss distribution for the multi-factor model estimated is used as the benchmark. Then, under the infection model, its VaR is computed by varying the magnitude of q until its value equals that of benchmark. We call the value of this solution as q*.

Secondly, the actual value of infectious default for each industry sector is determined as follows. Whereas q* is generally dependent on certain numbers of characteristics such as the distribution of exposures across sectors, correlation of default events, and default probabilities in its corresponding portfolio. According to Duellman (2006), the proxy of q* for every portfolio is the function of three systematic factors: the Herfindahl concentration index or HHI, the average probability of default, and the intra- and inter- correlation.

The HHI is calculated as the sum of squared relative exposure shares of the sectors in the portfolio. The intra- and inter- correlation is the weighted-average intra-sector and inter-sector asset correlation of all exposures weighted by the total exposure amounts to the individual sectors, respectively. The asset correlation can be estimated from the default correlation.

In this paper, the estimation of calibrated q is performed by running the regression between the values of q in each sector that equalize the 'infection model' unexpected loss percentage with that of 'true model' and their corresponding HHI, intra-correlation, and inter-correlation values and then using the coefficients from this regression equation as the weights to multiply with the actual three inputs in each 8 sector to come up with the values of calibrated q. This method is done within the limited amount of data environment.

Using the above calibrated q, we can estimate the result of unexpected loss percentage in the infection model for each sector. For HHI, we used market capitalization in stock market for 8 Thai industry sectors in SET to perform the concentration index. For intra correlation and inter correlation, earnings from 8 industry sectors are used to calculate correlation within and among sectors.

Second step: 'Infection Model' Buildup

Consider n identical bonds, but they are no longer independent. Let (Z_i, $i = 1, \ldots, n$) be random variables such that $Z_i = 1$ if bond i defaults and $Z_i = 0$ otherwise. Thus, the number defaulting is

$$N = Z_1 + Z_2 + \ldots + Z_n , \qquad \textbf{(11)}$$

In this model, the value of Z_i is determined as follows. For $i = 1, \ldots, n$ and $j = 1, \ldots, n$ with $j \neq i$, let X_i , Y_{ij} be independent Bernoulli random variables with

$$P[X_i = 1] = p$$

$$P[Y_{ij} = 1] = q$$

Then

$$Z_i = X_i + (1 - X_i)(1 - \prod_{j \neq i}(1 - X_j Y_{ji})) \qquad \textbf{(12)}$$

There are two channels in which the loan i, (or Z_i is equals 1), would turn into default: 'directly' and 'indirectly'. The first venue of default is in a straight-forward fashion: the customer i is just simply in the default mode and results in the loan i in default.

Alternatively, under the second venue of default, even if the customer i is still in the good shape, the customer j, who is in financial distress (X_j equals 1), can also infect the customer i to result in the loan i turning default.

For default distribution, let $F(n, k, p, q)$ denote the probability mass distribution of the random variable N, defined by (13) with Z_i given by (12), i.e.

$$F(n, k, p, q) = P[N = k] \qquad \textbf{(13)}$$

The distribution function F defined by (9) is given by

$$F(n, k, p, q) = C_k^n \alpha_{nk}^{pq}$$

Where $\alpha_{nk}^{pq} = p^k (1-p)^k (1-q)^{k(n-k)} + \sum_{i=1}^{k-1} C_i^k p^i (1-p)^{n-i} (1-(1-q)^i)^{k-i} (1-q)^{i(n-k)}$

The expected value is $E[N] = n(1-(1-p)(1-pq)^{n-1})$

3.3 Asymptotic Single Risk Factor (ASRF) Framework

Based on Basel Committee on Banking Supervision (2005)**,** the ASRF formula for computation of capital for financial institutions is as follows:

Correlation (R_{COR}) = 0.12×(1-EXP(-50×*PD*))/(1-EXP(-50))

+0.24×[1-(1-EXP(-50×*PD*))/(1-EXP(-50))]

Maturity Factor

Adjustment (*b*) = (0.11852-0.05478×ln(*PD*))^2

Percentage of Capital (*K*) = [*LGD* × N[(1-*R*)^-0.5 ×G(*PD*)+(*R*/(1-

R))^0.5×G(0.999)]-*PD*×*LGD*] ×(1-1.5×*b*)^-1×(1+(*M*-2.5) ×*b*)

Risk Weight Asset (*RWA*) = *K*×12.5×*EAD*

where

EXP = exponential function

ln(*x*) = the natural logarithm

N(*x*) = cumulative distribution function for a standard normal random variable

G(*z*) = the inverse cumulative distribution function for a standard normal random variable

This paper utilized the ASRF to calculate unexpected loss for 8 industry sectors in Thailand. For PD, we used NPL data from Bank of Thailand database from 2003 to 2009. For LGD, we followed the recommendation from Altman et al (2004). Finally, for EAD, the asset sizes for each industry are used in the analysis.

3.4 The diversification adjustment Model

This approach originates from the idea of utilizing the available information in the form of the single-factor capital to make some adjustment to come up with the approximation of the so-called ‘true model’ or multi-factor model capital. The approach will be divided into two steps:

First step: the derivation of single-factor sectoral capital:

In the measurement of risk level to set aside the amount of capital, the common approach currently is the Merton-model. The idea of this approach is viewed from the corporate sustainability viewpoint under the premise that the value of liability cannot exceed that of asset. To be more specific, obligor j defaults when a continuous random variable Y_j, which describes its creditworthiness, falls below a given threshold at the given horizon. If we denote by PD_j the obligor’s (unconditional) default probability and assume that the creditworthiness is standard normal, we can express the default threshold by $N^{-1}(PD_j)$.

Consider a credit model with K sectors where each of these sectors represents each business industry. For each obligor j in a given sector k, the credit losses at the end of the horizon (say, one year) are driven by a single-factor Merton model.

The creditworthiness of obligor j is driven by a single systemic factor:

$$Y_j = \sqrt{\rho_k} Z_k + \sqrt{1-\rho_k}\,\varepsilon_j \qquad \textbf{(14)}$$

where Z_k is a standard Normal variable representing the systemic factor for sector k, and the ε_j are independent standard Normal variables representing the idiosyncratic movement of an obligor’s creditworthiness. Whereas in the Basel II model all sectors are driven by the same systemic factor Z, here each sector can be driven by a different factor.

Let assume further that the systemic factors are correlated through a single macro-factor, Z

$$Z_k = \sqrt{\beta} Z + \sqrt{1-\beta}\,\eta_k \quad k = 1, \ldots, K \qquad \textbf{(15)}$$

where η_k are independent standard Normals. For simplicity we have assumed a single correlation parameter for all the factors (as we seek a simple parametric solution). Later, we allow for this parameter β to be more generally an average factor correlation for all the sectors.

For ease of notation, assume that for obligor j has a single loan with loss given default and exposure at default given by LGD_j, EAD_j respectively. As shown in Gordy (2003), for asymptotically fine-grained sector portfolios, the stand-alone α-percentile portfolio loss for a given sector k, $VaR_k(\alpha)$, is given by the sum of the individual obligor losses in that sector, when an α-percentile move occurs in the systemic sector factor Z_k :

$$VaR_k(\alpha) = \sum_{j \in Sectork} LGD_j EAD_j N\left(\frac{N^{-1}(PD_j) - \sqrt{\rho_k} z^{\alpha}}{\sqrt{1-\rho_k}}\right) \quad \textbf{(16)}$$

where z^{α} denotes the α-percentile of a standard normal variable.

Consider with Basel II capital rule, we define the stand alone capital for each sector, $C_k(\alpha)$ to cover only the unexpected loses. Thus, $C_k(\alpha) = VaR_k(\alpha)$ - EL_k, where $EL_k = \sum_{j \in Sectork} LGD_j EAD_j PD_j$ are the expected sector loss. The capital for sector k can then be written as

$$C_k(\alpha) = \sum_{j \in Sectork} LGD_j EAD_j \left[N\left(\frac{N^{-1}(PD_j) - \sqrt{\rho_k} z^{\alpha}}{\sqrt{1-\rho_k}}\right) - PD_j \right] \quad \textbf{(17)}$$

Under Basel II, or equivalently assuming perfect correlation between all the sectors, the overall capital is simply the stand-alone capital for all individual sectors (for simplicity, we omit the parameter α hereafter)

$$C^{1f} = \sum_{k=1}^{K} C_k \quad \textbf{(18)}$$

Second step: Adjustment to derive for the approximation of the multi-factor model capital

To come up with the amount of multi-factor model capital, we define the ***diversification factor*, *DF*,** as the ratio of the capital computed using the multi-factor model and the stand-alone capital $DF = C^{mf} / C^{1f}$, $DF \leq 1$.

To approximate *DF*, by a scalar function of a small number of intuitive parameters, we need this would allow us to express the (diversified) economic capital as a function of of a small number of intuitive parameters.

$$EC^{mf} \approx DF(.) \times \sum_{k=1}^{K} EC_k \quad \textbf{(19)}$$

Let us now first motivate the parameters used for this approximation. We can think of diversification basically being a result of to two sources. The first one is the correlation between the sectors. Hence, a natural choice for a parameter in our model is the correlation of the systemic sector factors Z_k.

The second source is the relative size of various sector portfolios. Clearly, one dominating very large sector leads to high concentration risk and limited diversification. The former is represented by defining the ***capital diversification index (CDI)*** as the sum of squares of the *capital weights* in each sector

$$CDI = \frac{\sum_{k=1}^{K} {EC_k}^2}{(C^{1f})^2} = \sum_k {w_k}^2 \qquad \textbf{(20)}$$

with $w_k = C_k / C^{1f}$ the contribution to one-factor capital of sector *k*. The *CDI* is simply the well-known Herfindahl concentration index applied to the stand-alone capital of each sector (rather than to the exposures, as is more commonly used). Intuitively, it gives an indication of the portfolio diversification across sectors (not accounting for the correlation between them).

For example, in the two-factor case, the *CDI* ranges between 0.5 (maximum diversification) and one (maximum concentration). The inverse of the *CDI* can be interpreted as an "effective number of sectors" in the portfolio, from a capital perspective. Note that one can similarly define the Herfindahl index for sector or counterparty exposures (*EADs)*, which results in a measure of concentration in terms of the size of the portfolio (and not necessarily the capital).

For the latter, the indication of the portfolio diversification across sectors is captured by

$$\bar{\beta} = [N_2(N^{-1}(PD_i), N^{-1}(PD_j), \rho\beta) - \rho^2] / [N_2(N^{-1}(PD_i), N^{-1}(PD_j), \rho) - \rho^2]$$

For the approximation, if credit losses were normally distributed, the credit capital at a given confidence interval, $C^{mf} = DF^N(CDI, \bar{\beta})C^{1f}$ with $DF^N = \sqrt{(1-\bar{\beta})CDI + \bar{\beta}}$

In this paper, PD is estimated by using NPL from Bank of Thailand database as in section 3.3 to do the analysis. The intra- and inter-asset correlation is estimated in the similar way as those from section 3.2.

4. Result and Analysis

In the first section, the descriptive statistics for each industry sector in Thai economy is demonstrated. Then, the comparison of the sector-by-sector unexpected loss percentage or capital ratio between ASRF and BET models against that of multi-sector Monte Carlo simulation or 'true model' is performed.

In the second section, the unexpected loss percentage or capital ratio result is estimated when incorporating infection model into the analysis. The calibrated q from the relationship with HHI, intra-correlation, and inter-correlation is used as the input to compute the outcome under infection model. In addition, we explore the role of q on the probability default frequency diagram for selected sectors in the Appendix.

Finally, we used the 'capital-diversification' adjustment from Cespedes (2005) to compute the unexpected loss percentage or capital ratio for the portfolio to compare with those of other three previous models.

4.1 ASRF and BET against Multi-Sectoral Monte Carlo Simulation or 'true model'

The descriptive statistics for each industry sector in the Thai economy, as of Quarter 1 in 2009, is shown to capture the outlook of Thai corporate market structure.

Table 1: Market Capitalization and HHI Index by Sectors of Thai Economy

Sector Name	Market Cap (Million Baht)	Percentage of Total	HHI Index
1. Natural Resources	1,482,113	35.1%	25.1%
2. Financial Services	925,924	21.9%	13.2%
3. Service	377,812	9.0%	18.6%
4. Real Estate and Construction	509,673	12.1%	27.3%
5. Technology	524,824	12.4%	29.7%
6. Agriculture & Foods Industry	175,942	4.2%	22.4%
7. Consumer Goods	50,910	1.2%	15.3%
8. Raw Material and Industrial Goods	171,995	4.1%	20.9%

Note that the biggest sector in terms of market capitalization size is natural resources sector primarily consisting mainly of energy companies. On the other hand, in terms of degree of concentration or Herfindahl Index, technology sector, mainly composed of communications and electrical products companies, has the highest HHI Index.

Table 2: Details on Square Error in Unexpected Loss Estimation for ASRF and BET Model with respect to 'True' Model by Sector

Sector Name	True Model Unexpected Loss (%)	ASRF Unexpected Loss (%)	BET Unexpected Loss (%)	Square Error in Percentage	
				ASRF	BET
1. Natural Resources	8	8.4	6	0.2	6.3
2. Financial Services	1	4.7	0.9	1,354.2	1.0
3. Service	6	7.9	5.2	10.0	1.7
4. Real Estate and Construction	11	10.6	12	0.2	0.8
5. Technology	12	9.1	7	5.9	17.4
6. Agriculture & Foods Industry	15	10.0	9.1	11.1	15.5
7. Consumer Goods	14	8.6	6.3	14.8	30.3
8. Raw Material and Industrial Goods	1	9.1	7.2	-	-

For the portfolio model performance measured sector by sector, in general, the BET model better captures the true model than the ASRF framework does, especially for financial services, service, and raw material and industrial goods sector.

According to Table 2, it illustrates the square error of unexpected loss estimation for both approaches with respect to the benchmark of the true model.

4.2 The Infection Model Analysis

According to Davis and Lo (1999), the ‘infection model’ is used to estimate the unexpected loss percentage or capital ratio by using the calibrated q from the relationship with HHI, intra-correlation, and inter-correlation as the input.

This section will start with the process of ‘infectious defaults’ determination or q-calibrated by using its relationship with HHI, intra-correlation, and inter-correlation used to compute the input under infection model. Once the calibrated q is determined, the infection model is simulated to estimate unexpected loss percentage or capital ratio of the ‘true model’. Finally, we compare the sector-by-sector model performance of ASRF formula, BET, and infection model by ranking on the square error in unexpected loss or capital estimation across sectors against the ‘true’ model.

4.2.1 Determination of ‘infectious defaults’ or q-calibrated

Using the characteristic of portfolio to estimate the value of q* is performed through the relationship of the infectious defaults with HHI, intra-correlation, and inter-correlation. Then, these estimated values of q* are used as the input to compute the outcome from the infection model. The diagrams of each pair relationship plot are as follows.

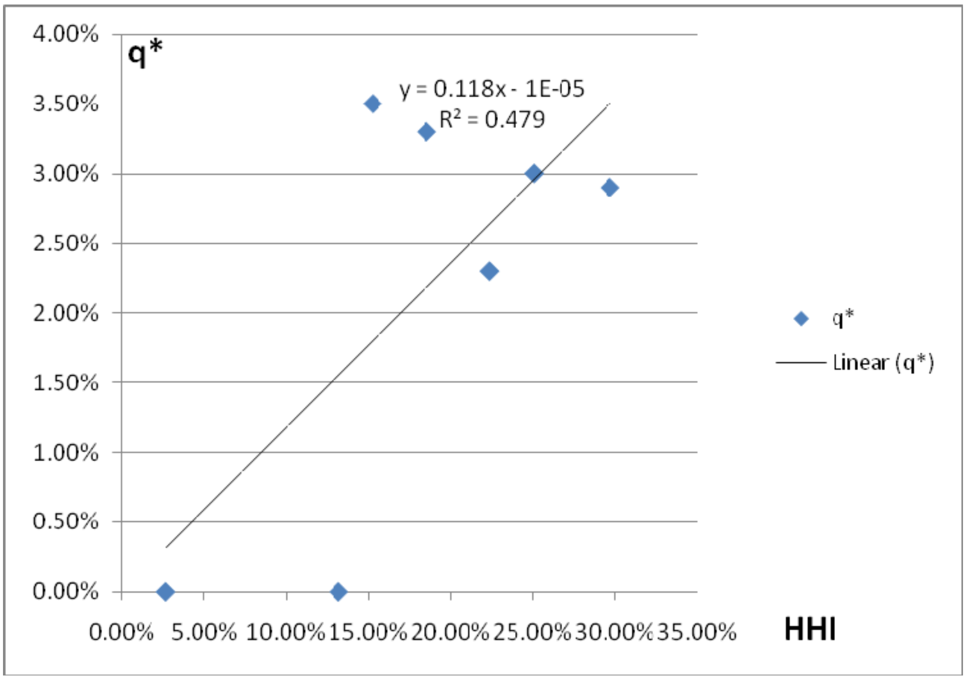

Figure 1: The relationship between HHI and q*

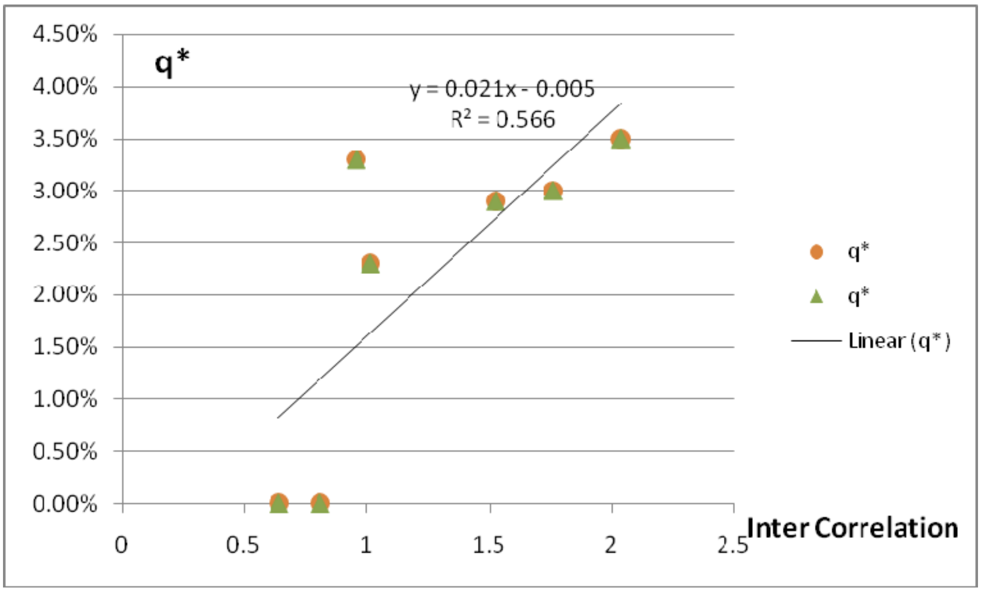

Figure 2: The relationship between inter-correlation and q*

From the relationship of HHI and inter-correlation for the whole portfolio vs. q*, separately, there is the relatively strong causal relationship between these two pairs of variables with the R-square in the range of between 40 to 60 percent, as illustrated in figure 1 and 2. Furthermore, the empirical results also show the priori signs as expected: the higher degree of concentration (HHI) and of inter-correlation, the higher degree of infectious defaults or q*. Numerically, with the increase in approximately 100 percent of HHI index and inter-correlation, this would result in an increase in approximately 11 percent and 2 percent of calibrated value of infectious defaults.

For the multivariate relationship among HHI, intra-correlation, and inter-correlation for the whole portfolio toward of calibrated value of infectious defaults, the result is illustrated in Table 3.

Table 3: Regression Results

Regressor	Intra	Inter	HHI
Estimate	-0.032	0.008	0.075
Standard Error	.028	.011	.050

The adjusted R-square for the equation, demonstrating the relationship between q* and (1) intra-correlation denoted by Intra (2) inter-correlation abbreviated by Inter, and (3) Herfindahl Index of market capitalization for each sector denoted by HHI, equals 60 percent.

Given this relationship, these three coefficients of portfolio characteristic variables multiplied with the value of intra-correlation, inter-correlation, and Herfindahl Index for each sector result in the value of infectious defaults or q to use an input to estimate the unexpected loss or capital for each sector in the infection model.

From the statistical relationship, the degree of fitness or explanatory power of the regression equation to the data set based on R-square and adjusted R-square is relatively high in the range of 60-80 percent. However, the significance level in explanatory power to infectious defaults of each individual variable in the regression equation is in medium range for intra-correlation and Herfindahl index, and relatively low degree of significance level for inter-correlation. The estimated result of infectious defaults and their percentage error with true model are demonstrated as follows.

Table 4: q estimated from Regression vs. q from infection with true model simulation

	q (Regression)	q*	Percentage Error
Sector 1	3.40%	3.00%	13%
Sector 2	0.01%	0.00%	0%
Sector 3	2.17%	3.30%	-34%
Sector 4	0.77%	0.00%	0
Sector 5	3.62%	2.90%	25%
Sector 6	2.34%	2.30%	0
Sector 7	3.04%	3.50%	2%
Sector 8	2.03%	0.00%	-13%

From Table 4, the estimated q from the three characteristic portfolio variables can reasonably become the proxy for the q value in the sense that it makes the unexpected loss (UL) or capital ratio in each sector close with that under the true model. The average error is approximately -2 percent from the exact q* that matches infection model with the 'true model'. For the model performance by sector, infection model is compared with previous three models as follows.

Table 5: True Model, ASRF, BET, and Infection Model Result

Sector Name	PD (%)	Intra-correlation (%)	Inter-correlation (%)	Unexpected Loss (%)			
	7 Year Historical	7 Year Historical	7 Year Historical	Multi-Sector Monte Carlo (True Model)	ASRF	BET	Infection Model
1. Natural Resources	9.36	31	25	8	8.37	6	13
2. Financial Services	1.49	81	9.2	1	4.68	0.9	0.9
3. Service	8.04	34	13.7	6	7.90	5.23	5.4
4. Real Estate and Construction	19.75	22	11.6	11	10.56	12	12
5. Technology	11.73	29	21.8	12	9.09	7	26
6. Agriculture & Foods Industry	15.88	39	14.5	15	10.01	9.1	15
7. Consumer Goods	10.12	26	29.1	14	8.62	6.3	6.4
8. Raw Material and Industrial Goods	11.73	75	31	1	9.09	7.2	7.2

According to table 5, under financial services sector, the result shows that the unexpected loss or UL from ASRF model is much higher than that of 'true model' does. The reason is that the ASRF does not take inter-correlation into account on the formula, in which its value is the lowest among all sectors due to the fact that financial institutions can decide not to lend or doing business when any particular industry is not profitable, or when the whole economy is in severe recession. On the other hand, they can choose to expand their lending when there is a sign that the economy is likely to be brighter. In other words, the opportunity to examine throughout all sectors in the economy results in an edge in doing business for financial service sector over others. As a result, this would render the number of defaults in the model not so high.

Secondly, the unexpected loss for technology sector under infection model is much higher than that under 'true model' and others. The reason is that the Herfindahl Index for the sector is very high due to its market structure concentrated in electronics and communications companies, resulting in the very high value of q and, henceforth, large magnitude of unexpected loss.

Finally, notice that for raw material and industrial goods sector, while, under the true model, the unexpected loss percentage or capital ratio is only 1 percent, those under ASRF, BET, and infection models are in the range of 7.2 and 9.1 percent. This is due to the absent of the direct role of inter-correlation in the latter three models, while raw material and industrial goods sector is the sector that supplies the material for production (considered as the indispensible ingredients) to other sectors. As a result, the external shock to this sector is in part alleviated by those sectors that depend on this upstream sector. To be more specific, this sector supplies the raw and material to at least other three sectors including automobile, petrochemical, and steel for consumer goods. As a result, the degree of hedging loss from economic downturn is very strong in this particular sector.

To facilitate the comparison among these three models, Table 6 illustrates the ranking for the performance of these models in terms of the sum of square error across sectors and standard deviation of square error across sectors. The best performing model regarding to these two aspects is 'infection model' (with calibrated q), followed intimately with BET model, and ASRF formula came remotely last.

Table 6: Ranking on Error in Unexpected Loss Estimation across Sectors against 'True' Model

	Sum of Square Error across Sectors (%)	Standard Deviation of Square Error across Sectors (%)	Ranking
Infection Model	40	1349	1
BET	39	1355	2
ASRF	79	2292	3

4.3 'Diversification-Adjustment' Model according to Cespedes et al (2005)

The result of the 'Diversification-Adjustment' model classified by sector is illustrated in Table 7.

Table 7: Result of 'Diversification-adjustment' Model by Sector

Sector Name	PD (%)	ρ_k	C_k (%)	$\sum_{k=1}^{K} C_k$ (%)
1. Natural Resources	9.36	.7	4.21	0.176958
2. Financial Services	1.49	.01	0.01	0.000001
3. Service	8.04	.39	1.44	0.020767
4. Real Estate and Construction	19.75	.01	0.07	0.000052
5. Technology	11.73	.01	0.05	0.000021
6. Agriculture & Foods Industry	15.88	.01	0.06	0.000036
7. Consumer Goods	10.12	.55	2.93	0.085557
8. Raw Material and Industrial Goods	11.73	.42	2.12	0.045047
Total			10.88	0.003284375

This contributes to the capital diversification index (CDI) of 0.277. The diversification factor (DF factor), derived from CDI and $\bar{\beta}$, equals 96.3 percent. As the one-factor model capital ratio is multiplied by DF factor, it results in the multi-factor capital ratio of 10.48%, as outlined in Table 8.

Table 8: 'Diversification-adjustment' Model Decomposition

C_{1f}	CDI	$\bar{\beta}$	DF	C_{mf}
10.88%	**0.277**	**90%**	**96.3%**	**10.48%**

For the overall portfolio performance of all models illustrated in Table 9, the infection model is the best in this aspect, following by ASRF formula, BET model, and 'diversification-adjustment' model, respectively.

Table 9: True Model, ASRF, BET, Infection Model, and 'Diversification-Adjustment' Model Result

Sector Name	Unexpected Loss (%)				
	Multi-Sector Monte Carlo (True Model)	ASRF	BET	Infection Model	'Diversification-adjustment' Model
1. Natural Resources	8	8.37	6	13	4.21
2. Financial Services	1	4.68	0.9	0.9	0.01
3. Service	6	7.90	5.23	5.4	1.44
4. Real Estate and Construction	11	10.56	12	12	0.07
5. Technology	12	9.09	7	26	0.05
6. Agriculture & Foods Industry	15	10.01	9.1	15	0.06
7. Consumer Goods	14	8.62	6.3	6.4	2.93
8. Raw Material and Industrial Goods	1	9.09	7.2	7.2	2.12
All	7.93	8.55	5.73	8.50	10.48

Table 10: Ranking Error in Unexpected Loss Estimation of All Portfolios against 'True' Model on Portfolio Level

Percentage Error of Capital Ratio at portfolio level	ASRF	BET	Infection Model	'Diversification-Adjustment' model
Percentage Error	7.4%	-27.7%	7.2%	32.2%

Comparatively, ASRF and BET model yield unexpected loss estimation with the error percentage equaling 7.4% and -27.7%, respectively, as shown in Table 10. On the other hand, the infection model, with infectious defaults or q determined by the relationship with HHI, intra-correlation, and inter-correlation, provides us with unexpected loss estimation with the error percentage equaling 7.2%.

Finally, as the benchmark, the sector level of 'true model' or multi sector Monte Carlo on capital ratio is illustrated in Table 11. This is derived from the weighted sector capital ratio in all sectors to come up with the capital ratio of 7.93 percent

Table 11: 'True Model' or Multi Sector Monte Carlo on Capital Ratio

Sector	True Model Capital (%)	True Model Loss (Million Baht)	True Model Portfolio Capital (%)
1. Natural Resources	8	4,140	7.93
2. Financial Services	1	106	
3. Service	6	935	
4. Real Estate and Construction	11	1,631	
5. Technology	12	1,461	
6. Agriculture & Foods Industry	15	1,382	
7. Consumer Goods	14	355	
8. Raw Material and Industrial Goods	-	-	

5. Conclusion

For the overall portfolio model performance, we found that, under 'the infection model' by Duellmann (2006), it can estimate the unexpected loss or capital under the multi-sector Monte Carlo simulation (true model) fairly well with Thai historical data between 2003 and 2009. On the other hand, 'diversification-adjustment' approach by Cespedes et. al. (2005) can estimate the unexpected loss or capital under the multi-sector Monte Carlo simulation (true model) relatively poor with Thai historical data.

In comparison with both stated approaches, the overall portfolio model performance of Asymptotic Risk Factor model (ASRF) and Moody's Binomial Expansion Technique (BET) in estimating unexpected loss of the true model lies in between those approaches, with ASRF closes tightly with that of infection model.

On the other hand, in terms of the sector-by-sector model performance, those of infection model, BET, ASRF, and 'diversification-adjustment' approach are from best to worst, respectively.

Appendix

The Analysis of Infectious Defaults for Selected Sectors

The appendix illustrates the distributions of default frequency of infection model where its unexpected loss (UL) matches with that of multi factor model on different sector. Their probability distributions are illustrated as follows.

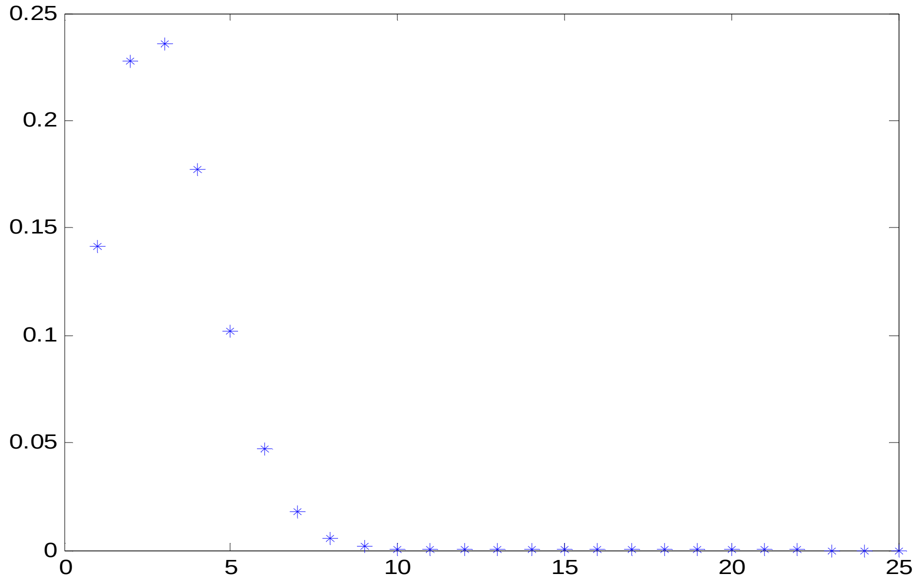

Figure 3: BET (Sector 1)

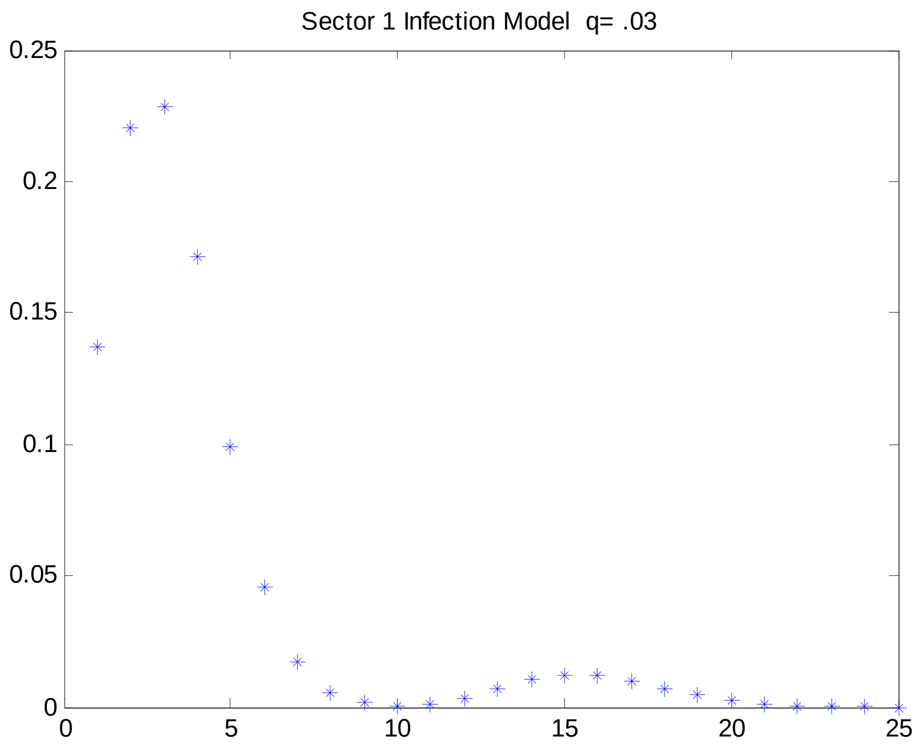

Figure 4: Infection Model (Sector 1)

Under natural resources sector, the BET model generates the lower value of unexpected loss than the 'true model' does. As a result, we need an infectious probability or q to elevate the number of default to match with the 'true' model. In figure 4, the number of defaults ranging from 10 to 20 rises from zero to approximately 1%.

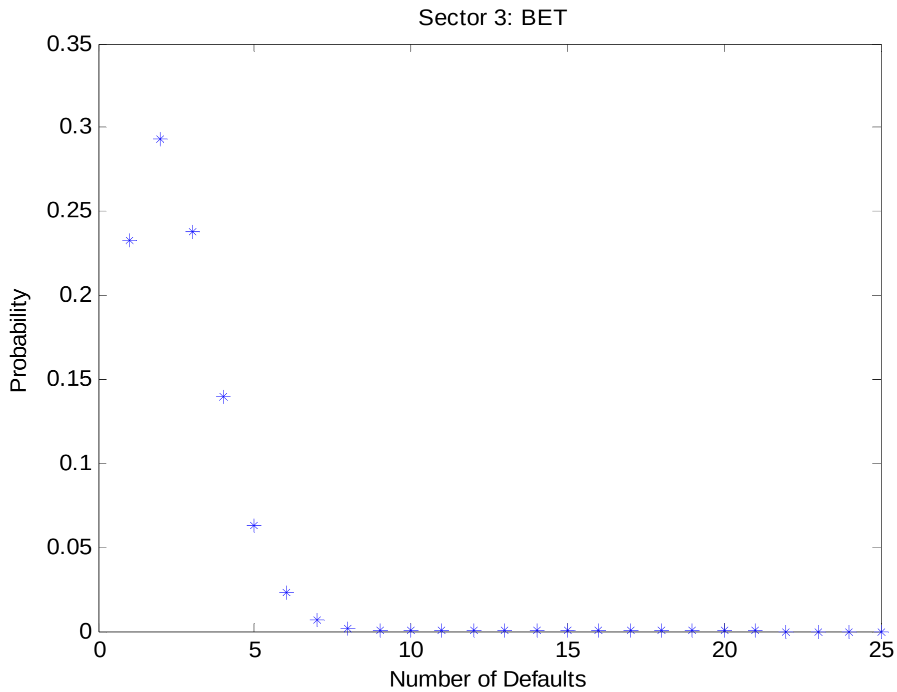

Figure 5: BET Model (Sector 3)

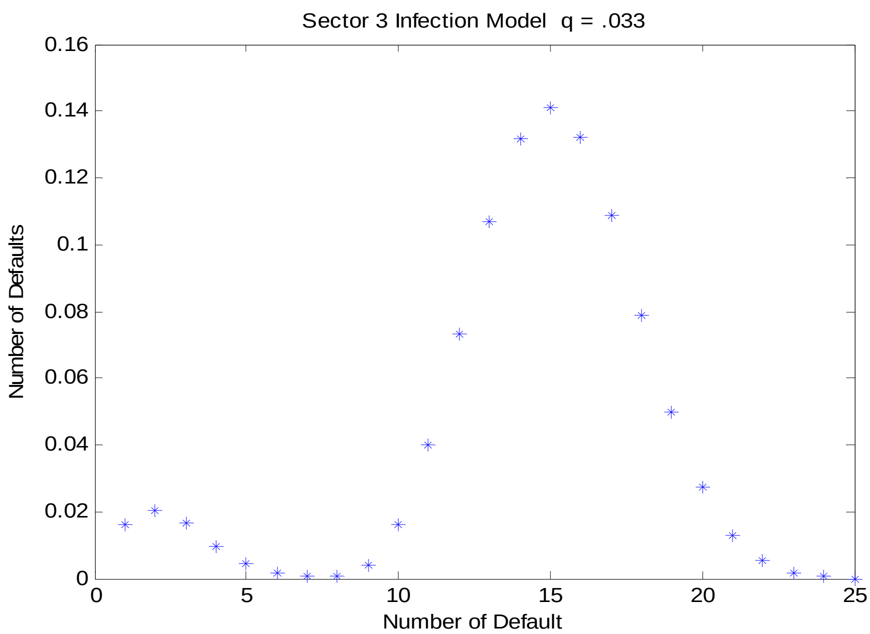

Figure 6: Infection Model (Sector 3)

For service sector, the BET model is slightly lower than 'true' model. As a result, we need infectious defaults or calibrated q value to increase the unexpected loss. Therefore, this would result in the flattening of default frequency distribution (expanding the range of default rate, but it also lowers the frequency on each default rate.

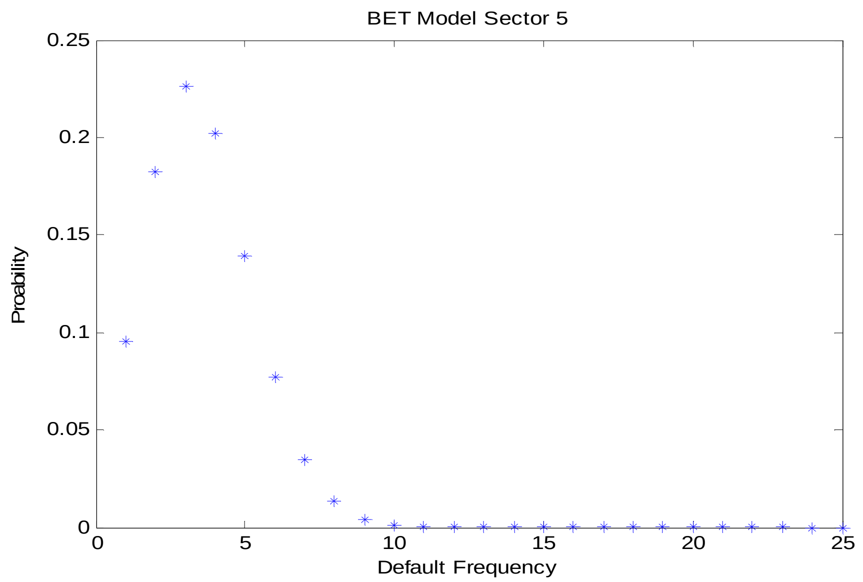

Figure 7: BET Model (Sector 5)

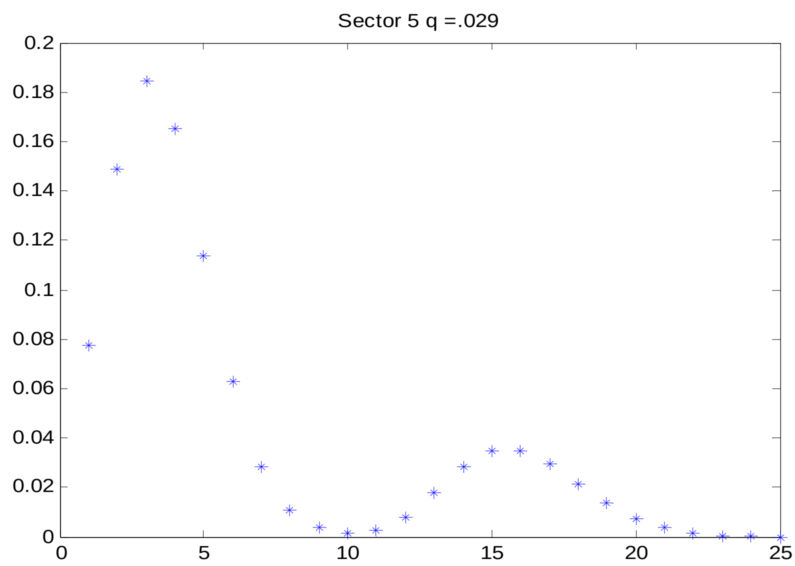

Figure 8: Infection Model (Sector 5)

For technology sector, the calibrated infectious default from technology sector is higher than the one that equalizes the capital ratio with that of true model. This is partly due to the highest value of Herfindahl Index or HHI among all eight industries. From figure 8, the bimodal curve with the relatively high number and high probability of default frequency result in the high unexpected loss percentage.

Note that for agriculture sector, the infection model most precisely matches that of true model.

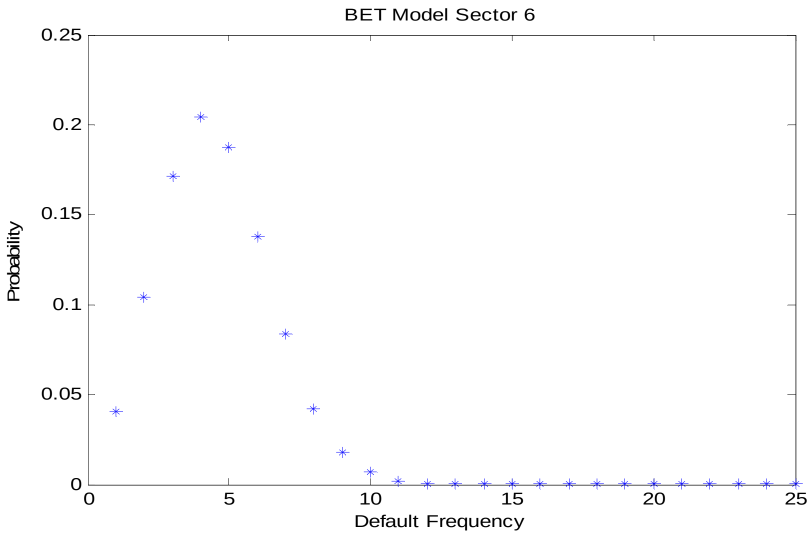

Figure 9: BET Model (Sector 6)

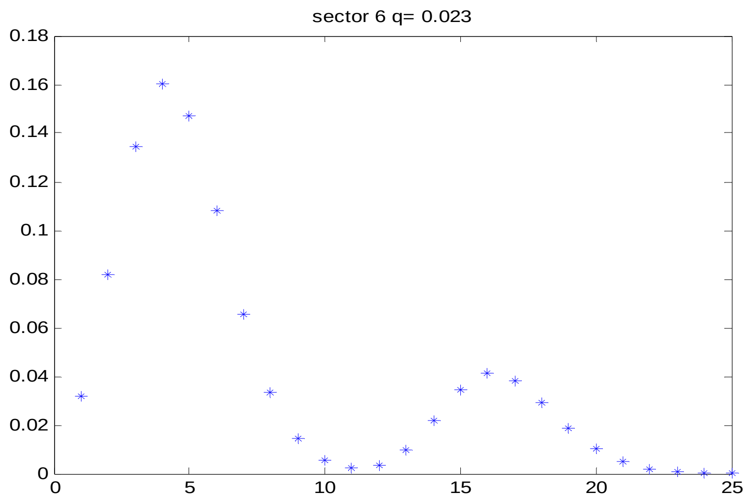

Figure 10: Infection Model (Sector 6)

For consumer goods sector, with the very low concentration index or HHI and intra-correlation, it results in the low value of infectious defaults. This would therefore generate the lower unexpected loss percentage than the 'true model'.

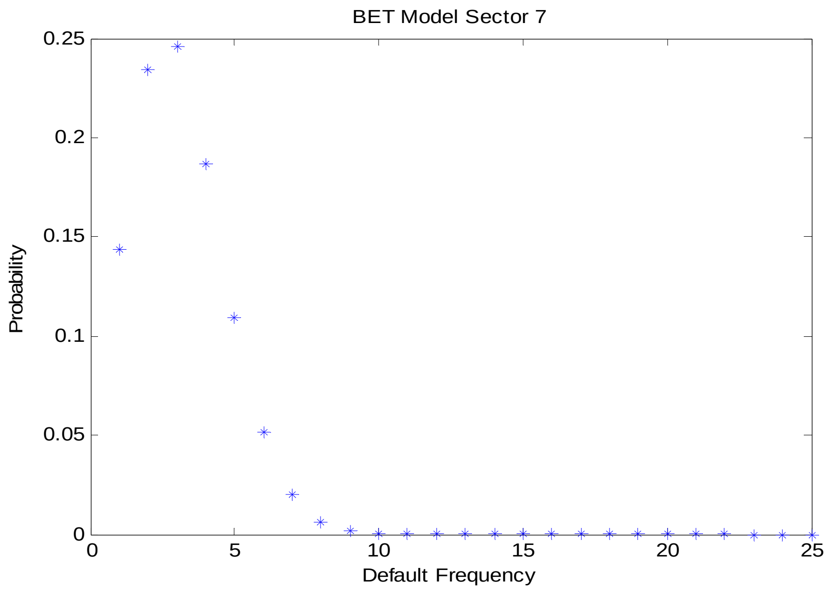

Figure 11: BET Model (Sector 7)

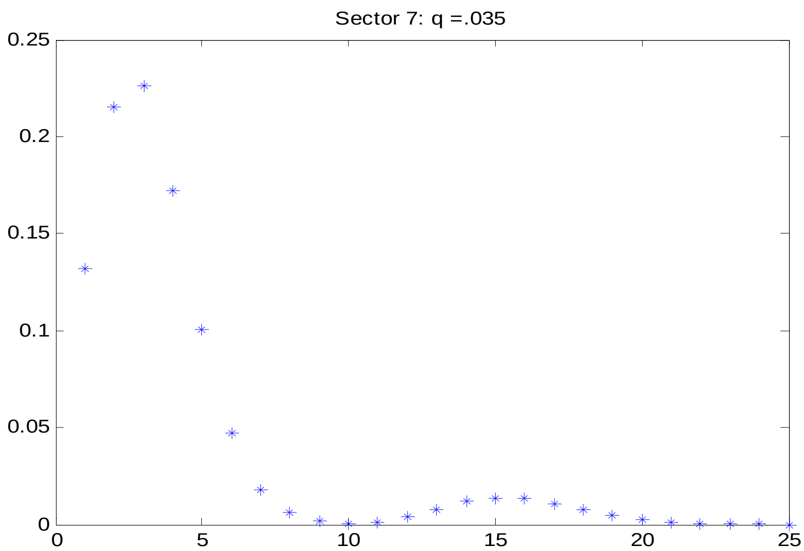

Figure 12: Infection Model (Sector 7)

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