The Role of Bank Capital and The Transmission Mechanism of Monetary Policy

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Abstract

This paper studies the transmission mechanism of monetary policy in the presence of an endogenous role for bank capital. The basic framework is a standard Dynamic New Keynesian model modified so that both firms and banks face endogenous financial frictions in obtaining external funds. The model exhibits an unconventional ‘bank capital’ channel in which monetary policy also affects aggregate economic activity via its effect on bank capital. The simulation results highlight a financial accelerator effect in that the endogenous evolution of bank capital, and its dynamic interplay with that of entrepreneurial net worth and asset price, operate to amplify and propagate the effect of a monetary shock in the macroeconomy.

Keywords: bank capital, monetary policy, financial accelerator effect

JEL Classification: E30, E44, E50, G21

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1 Introduction

The goal of this paper is to study the transmission mechanism of monetary policy in the presence of an endogenous role for bank capital. This is motivated by the observation that conventional macro models for monetary policy abstract completely from the role of bank capital. This consensus practice would be a justifiable simplifying assumption only if one of the following conditions holds; 1) an unexpected monetary shock does not affect bank capital, or 2) if it does, changes in the dynamics of bank capital must have no major effect on that of other important aggregate macroeconomic variables.

One of the main functions that banks perform is the transformation of securities with short maturities, offered to depositors, into securities with long maturities that borrowers desire (Freixas and Rochet, 1997). This maturity mismatch on banks’ balance sheets implies that lending rates are relatively stickier compared to deposit rates in response to unanticipated aggregate shocks. Consequently, as the risk free rate rises unexpectedly, banks’ interest rate cost rises faster than their interest rate revenue, thereby depleting their inside capital. This invalidates the prior first condition; an unexpected monetary shock can theoretically affect bank capital.

Turning to the second condition, there are many empirical findings which lend support to the importance of the role of bank capital in constraining bank lending and aggregate economic activities. Amongst others, Bernanke and Lown (1991), Furlong (1992) and more recently, Peek and Rosengren (1997) and Ito and Sasaki (1998) find that the capital position of banks has had positive and statistically significant effects on bank lending. Moreover, Hubbard et al. (2002) find that higher bank capital lowers the rate charged on loans, even after controlling for borrower characteristics, other bank characteristics and loan contract terms. Given these findings, the second condition is also violated. Thus excluding bank capital from a model’s dynamics can distort our understanding of the monetary policy transmission mechanism.

The basic framework employed in this paper is an extension of the financial accelerator model of Bernanke et. al (1999). Here I propose the ‘double’ costly state verification (Double CSV) approach as the principal modification.1 It is ‘double’ because, in addition to firms, banks also face endogenous financial frictions in obtaining external funds. This implies that a wedge between the internal and external cost of funds exists, thereby motivating an endogenous role for firms’ and banks’ inside capital

1 The one-sided CSV problem was first introduced by Townsend (1979).
in the model. The novel feature of the approach is that, while retaining rigorous microfoundations found in the theory of banking literature, it is simple and tractable enough to be readily embedded into the otherwise standard Dynamic New Keynesian (DNK) model with price stickiness. This allows us to study, both qualitatively and quantitatively, how the dynamic evolution of bank capital operates to enrich the transmission mechanism of monetary policy by augmenting the dynamics of other aggregate macro variables, including aggregate investment, output and asset price.

The organisation of the paper is as follows. Section 2 discusses some related literature. Section 3 presents the partial equilibrium model of financial contract, the double CSV approach. By embedding the key equations derived in section 3 into the otherwise standard DNK model, section 4 presents a simulation analysis to demonstrate the operational mechanism of the monetary policy transmission implied by the model. Section 5 concludes the paper.

2 Related Literature

Bernanke et al. (1999) and Carlstrom and Fuerst (2001) examine the role of credit market frictions in business cycle fluctuations. Firms in these models face financial frictions in borrowing from banks. This allows monetary policy to have an independent effect on entrepreneurial net worth, the so-called balance sheet channel. However, banks do not need to hold any inside capital in equilibrium as they are assumed to have a perfectly diversified portfolio of bank loans. This implies that any idiosyncratic risk associated with firms’ investment return is completely diversified at the bank level and therefore is not passed on to ultimate depositors.

Van den Heuvel (2001) examines an explicit role for bank capital in a model of bank asset and liability management. Banks in his model hold their inside capital to satisfy exogenous capital adequacy regulations. This, together with a maturity mismatch on banks’ balance sheets, gives rise to a ‘bank capital’ channel in which monetary policy affects bank lending through its impact on bank capital. However, his model does not incorporate an explicit role for entrepreneurial net worth, and therefore does not exhibit its dynamic interplay with bank capital. Moreover, to the extent that his model abstracts from consumption, investment and aggregate demand effects relating to price stickiness, it is not a general equilibrium model.
The last set of papers endogenises both entrepreneurial net worth and bank capital. Bolton and Freixas (2000) analyse the transmission mechanism of monetary policy in the context where direct and indirect finance coexist. Given the presence of an exogenous capital requirement, asymmetric information on the value of bank capital implies an endogenous cost in raising outside equity capital. The monetary policy transmission implied by their model exhibits an amplification effect on bank lending through its effect on bank capital. Cantillo (1997) adopted the CSV approach to study coexistence between direct and indirect finance. In his model, both firms and banks face financial frictions in obtaining external funds, and therefore hold limited inside capital in equilibrium. However, as these models have only two periods, the issue of dynamics cannot be disentangled. Moreover, similar to Van den Heuvel (2001), they are not fully general equilibrium models.

Chen (2001) studies dynamic interaction between entrepreneurial net worth, bank capital and real economic activity by extending Holmstrom and Tirole’s (1997) model into a dynamic general equilibrium setting. The moral hazard problem both at the firm and bank levels is assumed in order to motivate an endogenous role for firms’ and banks’ inside capital. However, the model has no risk free rate and price stickiness, and therefore cannot be used to study the transmission mechanism of monetary policy.

3 The Partial Equilibrium Model of Double CSV

3.1 Basic Assumptions and the Structure of the Model

There are five types of agents in the economy; entrepreneurs, banks, households (depositors), retailers and the central bank. As the main focus of this paper is on the transmission of monetary policy, I shall abstract from the role of government and therefore fiscal policy in the model.

Entrepreneurs are assumed to be risk neutral and are the only type of agent in the economy with access to investment technology. A representative entrepreneur, say entrepreneur $i$, operates firm $i$. At the end of period $t$, firm $i$ purchases capital, $K^i_t$. Its unit price is given by $Q_t$. All capital is homogenous.
It is assumed that capital purchased at the end of period $t$ cannot be used in production until the end of period $t+1$. The gross rate of return from investing in capital is given by $\omega_{i,t+1}R_{t+1}^K$, where $R_{t+1}^K$ and $\omega_{i,t+1}$ are the non-idiosyncratic and idiosyncratic components of firm $i$’s rate of return to capital, respectively.\(^3\)

The random variable $\omega_{i,t+1}$ is assumed to be log normally distributed with mean one and variance $\tau^2$, and is independently and identically distributed (i.i.d.) across time and firms.\(^4\) Because the non-idiosyncratic component of return associated with period-$t$ capital investment, $R_{t+1}^K$, will not be realised until the end of period $t+1$, in addition to idiosyncratic risk, firm $i$ also encounters aggregate risk.

I assume that firm $i$ can borrow external funds from a representative bank, say bank $j$, to partially finance its capital investment. All financial contracts are assumed to have one period maturity. Following the CSV literature, the realisation of the idiosyncratic component of return, $\omega_{i,t+1}$, is private information and bank $j$ has to pay a verification cost in order to observe its value. This, as mentioned earlier, motivates entrepreneur $i$ to hold his inside capital as the existence of a verification cost drives a wedge between internal and external cost of funds. Moreover, following Krasa and Villamil (1992), I assume that the realisation of $\omega_{i,t+1}$ is privately revealed only to the agent who requests the CSV technology. This assumption is essential to the analysis since if all information could be made public ex post there would be no need for depositors to pay a verification cost to observe banks’ return on their portfolio of loans.

Denote firm $i$’s inside capital held at the end of period $t$ by $W_i^t$. Given that the total outlay of the investment is $Q_tK_i^t$, loans borrowed from bank $j$ is given by $L_i^t \equiv Q_tK_i^t - W_i^t$.

Using the fact that the total return from investing $Q_tK_i^t$ is $\omega_{i,t+1}R_{t+1}^K Q_t K_i^t$, firm $i$’s threshold value of $\omega_{i,t+1}$, $F_{i,t+1}$, is defined such that it satisfies the following equation.

\[
F_{i,t+1}R_{t+1}^K Q_t K_i^t = r_{i,t}^L L_i^t
\]

where $r_{i,t}^L$ is defined as the non-default loan rate associated with the loan contract between firm $i$ and bank $j$.

\(^3\)Thoughout the paper, the time subscript denotes the period in which the value of an underlying variable is realised.

\(^4\)Formally, the distribution of the random variable $\omega_{i,t+1}$ can be summarised as follows; $\ln \omega_{i,t+1} \overset{i.i.d.}\sim N(-\frac{1}{2}\sigma^2, \sigma^2)$, where $\sigma^2 = \ln(1 + \tau^2)$

\(^5\)Denote $F(\omega_{i,t+1})$ and $f(\omega_{i,t+1})$ as c.d.f. and d.f. of $\omega_{i,t+1}$, respectively, and let $h(\omega_{i,t+1}) \equiv \frac{f(\omega_{i,t+1})}{1 - F(\omega_{i,t+1})}$ be the hazard rate, the assumption that $\omega_{i,t+1}$ is log-normally distributed implies that the restriction $h(\omega_{i,t+1}h(\omega_{i,t+1})) > 0$ holds. This regularity condition is a relatively weak restriction as it is satisfied by most conventional distributions (Bernanke et al., 1999).
and bank $j$ signed in period $t$. When $\omega_{i,t+1} < \bar{F}_{i,t+1}$, firm $i$’s realised revenue from investing $Q_tK_{i,t}$ is insufficient to fulfil its loan contract with bank $j$. Then firm $i$ declares bankruptcy and faces liquidation. In contrast, when $\omega_{i,t+1} \geq \bar{F}_{i,t+1}$, firm $i$ does not go bankrupt as its realised return in period $t+1$ is sufficient to repay its debt obligation to bank $j$.

### 3.1.2 Banking Sector

Banks in this economy operate under a perfectly competitive environment. Similar to entrepreneurs, they are assumed to be risk neutral.

As commonly shown in the conventional one-sided CSV literature\(^7\), given that $\omega_{i,t+1}$ is $i.i.d.$ across firms, the idiosyncratic risk associated with each investment project is completely diversified in the infinitely large portfolio of bank loans, by virtue of the law of large number. Thus, depositors can be guaranteed an equivalently riskless rate of return and banks have no incentive to hold any inside capital. However, as argued by Krasa and Villamil (1992), the diversification would not be complete if the portfolio is of a finite size, in which case idiosyncratic risk associated with firms’ investment projects remains at the bank level and therefore is passed on to ultimate depositors. This gives an incentive for depositors to monitor banks.\(^8\)

In general, given that the size of bank loan portfolio is finite, the distribution of return within each bank becomes essential to the analysis.\(^9\) To maintain the model’s tractability, I assume that a bank can only lend to one entrepreneur.\(^10\) Although this assumption is obviously unrealistic, it is meant to avoid the equally unrealistic conclusion that banks can never collapse and its intermediation service can be carried out without any inside capital.

Bank $j$ can borrow external funds from a representative depositor, say depositor $m$, to partially finance its lending to firm $i$. Given the assumption that a bank can only lend to one entrepreneur, the idiosyncratic risk associated with firm $i$’s investment is passed on directly to bank $j$’s returns on its

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6. As $\omega_{i,t+1}$ is realised in period $t+1$, its value is contingent on the ex-post realisation of $R_{i,t+1}^K$.


8. Thus the CSV problem becomes two-sided. On the one hand, banks act as delegated monitors on firms’ investment projects. On the other hand, in the terminology of Krasa and Villamil (1992), depositors perform the role of ‘monitoring the monitor’.

9. For example, the contract term and the aggregation process depend on the distribution of an individual bank’s portfolio of risky loans.

10. This assumption is essentially tantamount to the case where a bank can finance multiple firms but the return on firms’ investment projects are assumed to be perfectly correlated within a bank but $i.i.d.$ across banks (Holstrom and Tirole, 1997 and Chen, 2001). It implies that idiosyncratic risk is fully diversified at the aggregate level, but not at the bank level.
lending. As firm i’s return on investment is private information, so is the return on bank j’s loans. Given the CSV problem at the bank level, depositor m has to pay a verification cost if he wishes to observe the return on bank j’s lending. This creates an external finance premium for bank j in obtaining external funds from depositor m, thereby motivating the bank to hold inside capital. So, the holding of bank capital in this model is a market-based, as opposed to a regulatory-based, requirement.

In period t, bank j which finances its lending to firm i \((L^i_t)\) by its own inside capital \((A^i_t)\) and deposits acquired from depositor m \((D^i_t)\) has the following balance sheet identity:\(^{11,12}\)

\[ L^i_t = D^i_t + A^i_t \]  

(2)

The verification cost that bank j has to pay in the event that firm i declares bankrupt is assumed to equal a proportion \(\theta^B\) of the realised gross return to firm i’s investment \((\theta^B \omega_{i,t+1} R^K_{t+1} Q_{t} K^i_t)\). In such event, bank j would receive the net liquidation revenue from firm i equivalent to \((1 - \theta^B)\omega_{i,t+1} Q_{t} R^K_{t+1} K^i_t\). Given this value, bank j’s threshold value of \(\omega_{i,t+1}, R^B_{i,t}\) is defined such that it satisfies the following equation.

\[ (1 - \theta^B)\omega_{i,t+1} Q_{t} R^K_{t+1} K^i_t = r^{D}_{t+1} D^i_t \]  

(3)

where \(r^{D}_{t+1}\) denotes the non-default deposit rate realised in period \(t + 1\) associated with the deposit contract between bank j and depositor m signed in period \(t\). When \(\omega_{i,t+1} \geq \omega^B_{i,t+1}\), the bank’s net revenue received from liquidating firm i is sufficient to fill the deposit contract. In contrast, when \(\omega_{i,t+1} < \omega^B_{i,t+1}\), the bank declares bankruptcy as its net liquidation revenue is insufficient to repay its debt obligation to the depositor.\(^{13}\)

\(^{11}\)Similar to loan contracts, I assume that all deposit contracts have only one period maturity.

\(^{12}\)Compared to the asset side of the balance sheet of a typical bank in reality, there are three important elements that this model lacks, namely short term bonds, cash and equipments. Because banks have to maintain the convertibility commitment with depositors at any point in time and that loans are a relatively illiquid kind of asset, banks have an incentive to hold short-term government bonds as a means of reducing their exposure to liquidity risk. However, the model at hand is not rich enough to accommodate the prevalence of liquidity risk.

Banks also have to hold cash or reserves (the liabilities of central banks) owing to their inability to forecast perfectly their payment flows and to arrange transactions in the interbank market throughout the day so as to maintain settlement balances constant at a balance greater than, or equal to, zero (Goodhart, 2000). As central banks have monopoly power in issuing ‘cash’, this gives them the power to set the short-term risk free rate. However, as emphasised by Woodford (2000), this power does not depend on the size of the banks’ cash holding. To simplify the analysis, I therefore assume that the size of cash holding by banks is negligibly small, i.e. approaching zero.

Lastly, I assume for simplicity that the value of equipments required in conducting banking service is zero.

\(^{13}\)Similar to \(\omega^B_{i,t+1}, \omega^D_{i,t+1}\), is realised in period \(t + 1\), implying that its value is contingent on the \textit{ex-post} realisation of \(R^K_{t+1}\).
3.1.3 Depositor (Household)

Depositors invest their savings by depositing their money with banks. Unlike entrepreneurs and banks, they are neutral to idiosyncratic risk but are averse to aggregate risk. This implies that aggregate risk inherited in the firms’ project has to be completely absorbed by entrepreneurs and banks. As can be seen from equations (1) and (3), unlike the non-default lending rate which is determined instantaneously once the loan contract is signed, the non-default deposit rate associated with the deposit contract signed in period $t$ will not be realised until period $t+1$. Consequently, as period $t+1$ arrives and aggregate risk associated with period-$t$ capital investment is uncovered, in response to a lower than expected realised return on non-idiosyncratic component of firm $i$’s investment return ($R_{i,t+1}^K < E_i(R_{i,t+1}^K)$), depositor $m$ will be compensated with a higher non-default deposit rate, so they are completely hedged against any plausible realisation of aggregate risk. Crucially, this assumption implies that the adjustment of the lending rate will be relatively stickier in response to a monetary shock compared to that of the deposit rate. As discussed in the Introduction, this proxies realistically the effect of having a maturity mismatch in the bank’s balance sheet.\footnote{As all financial contracts in the model have only one period maturity, a maturity mismatch in banks’ balance sheet cannot be explicitly modelled. The assumption on the risk profile of the agents is meant to implicitly deliver the effect of having a maturity mismatch in the model while, at the same time, maintaining the model’s tractability.}

Because the return on bank $j$’s portfolio is private information, depositor $m$ has to pay a verification cost if he wishes to observe its realisation. The verification cost is assumed to equal a proportion $\theta^D$ of the realised gross return to firm $i$’s investment ($\theta^D \varpi_{i,t+1} R_{i,t+1}^K Q_t K_i^t$). Given that I have defined the verification cost required to be paid by depositor $m$ and bank $j$, the ‘special’ role of bank $j$ as a delegated verifier can be summarised by the following assumption:

$$\left(\theta^D - \theta^B\right) \int_0^{\varpi_{i,t}} \varpi_{i,t} f(\varpi_{i,t}) d\varpi_{i,t} > (1 - \theta^B) \theta^D \int_0^{\varpi_{i,t}} \varpi_{i,t} f(\varpi_{i,t}) d\varpi_{i,t}$$

Intuitively, the left hand side (the right hand side) is the expected benefit (cost) from having banks in the economy. The expected benefit arises from the fact that bank $j$ can verify the outcome of the project relatively cheaper compared to depositor $m$, i.e. $\theta^B$ is sufficiently lower than $\theta^D$. On the contrary, the expected cost arises from the fact that depositor $m$ has to pay extra cost of monitoring ‘the monitor’ in certain states of the world. Thus, having bank $j$ as a financial intermediary dominates the one-sided financial contract between firm $i$ and depositor $m$ because the aggregate expected verification cost is...
To summarise the structure of the model, the next subsection illustrates its sequence of events.

3.1.4 The Sequence of Events

At the end of period $t$, entrepreneur $i$ chooses his optimal demand for capital ($K_i^t$). To partially finance his investment, he engages in a loan contract with bank $j$ and borrows $L_i^t$. The non-default lending rate associated with the loan contract ($r_{i,t}^L$) is simultaneously determined. To finance its lending to firm $i$, bank $j$ also engages in a deposit contract with depositor $m$ from whom it borrows $D_i^t$.

As time approaches the end of period $t+1$, the non-idiosyncratic component of firm $i$’s return on its period-$t$ investment ($R_{t+1}^K$) is realised. The deposit rate associated with the deposit contract signed in period $t$ ($r_{i,t+1}^D$) is then realised, which implies that the depositor is perfectly hedged against any plausible realisation of aggregate risk.

After the aggregate risk associated with period-$t$ financial contract is uncovered, firm $i$ decides on its optimal purchase of capital and its borrowing from bank $j$ in period $t+1$, $K_{i+1}^t$ and $L_{i+1}^t$, respectively. $r_{i,t+1}^L$ is simultaneously determined. The bank then borrows $D_{i+1}^t$ from the depositor.

Lastly, the idiosyncratic return to firm $i$’s investment in period $t$ ($\pi_{i,t+1}$) is realised. In the event that the entrepreneur (the banker) goes bankrupt, he pays whatever is left to his debtor and departs from the scene.

3.2 The Contract Term

Given that all firms and banks are subject to limited liability clauses, as shown by Gale and Hellwig (1985), optimal financial contracts with the presence of CSV become those of risky debt contracts. In this subsection, I study an optimal financial contract amongst firm $i$, bank $j$ and depositor $m$ and derive the firm’s optimal demand for capital. I proceed by finding the agents’ expected profit functions in subsection 3.2.1. Subsection 3.2.2 then uses these functions to solve for firm $i$’s optimal demand for capital.
3.2.1 The Agents’ Expected Profit Functions

Firm $i$’s Expected Profit Function: As can be seen from equation (1), when $\omega_{i,t+1} \geq \omega_{i,t+1}$, the return to firm $i$’s project is sufficient to repay its debt obligation to bank $j$, $r_{i}^{F}L_{i}^{j}$. It pays the contractual amount and retains $\omega_{i}Q_{t}R_{i}K_{i}^{j} - r_{i}^{F}L_{i}^{j}$. When $\omega_{i,t+1} < \omega_{i,t+1}$, firm $i$ declares a default, liquidates its assets, and retains nothing. Given that firm $i$’s opportunity cost of funds is the real risk free rate ($r_{t+1}^{F}$) its expected profit function in period $t+1$, conditional solely on the realisation of idiosyncratic risk ($\pi_{i,t+1}^{F}$), is given by\(^{15}\):

$$\pi_{i,t+1}^{F} = \int_{\omega_{i,t+1}}^{\infty} [\omega_{i,t+1}Q_{t}R_{i}K_{i}^{j} - r_{i}^{F}L_{i}^{j}]f(\omega_{i,t+1})d\omega_{i,t+1} - W_{i}^{F}r_{i}^{F}$$

(4)

Bank $j$’s Expected Profit Function: Bank $j$’s expected profit function depends in general on the relative value of $\omega_{i,t+1}$ and $\omega_{i,t+1}^{F}$. However, under the restriction that $\omega_{i,t+1} > \omega_{i,t+1}^{F}$, Appendix A shows that the expected profit function for bank $j$ in period $t+1$, conditional on the realisation of idiosyncratic risk, is given by:

$$\pi_{j,t+1}^{B} = \int_{\omega_{i,t+1}^{F}}^{\omega_{i,t+1}} [(1 - \theta^{B})\omega_{i,t+1}Q_{t}R_{i}K_{i}^{j} - r_{i}^{F}L_{i}^{j}D_{i}^{j}]f(\omega_{i,t+1})d\omega_{i,t+1}$$

$$+ [1 - F(\omega_{i,t+1}^{F})]r_{i}^{F}L_{i}^{j} - r_{i}^{F}D_{i}^{j}] - A_{i}^{F}r_{i}^{F}$$

(5)

Depositor $m$’s Expected Profit Function: Under the restriction that $\omega_{i,t+1} > \omega_{i,t+1}^{B}$, both firm $i$ and bank $j$ declare bankruptcy when $\omega_{i,t+1} < \omega_{i,t+1}^{B}$. So after paying the verification cost, depositor $m$ retains $(1 - \theta^{P})(1 - \theta^{B})\omega_{i,t+1}Q_{t}R_{i}K_{i}^{j}$. When $\omega_{i,t+1} > \omega_{i,t+1}^{B}$, bank $j$ does not go bankrupt and the depositor gets $r_{i}^{F}D_{i}^{j}$. Given the real risk free rate as the opportunity cost of funds, the depositor’s expected profit function in period $t+1$, conditional on the realisation of idiosyncratic risk, is given by:

$$\pi_{m,t+1}^{D} = \int_{\omega_{i,t+1}^{B}}^{\omega_{i,t+1}} [(1 - \theta^{P})(1 - \theta^{B})\omega_{i,t+1}f(\omega_{i,t+1})d\omega_{i,t+1}]R_{t}K_{i}^{j}$$

$$+ [1 - F(\omega_{i,t+1}^{B})]r_{i}^{F}D_{i}^{j} - D_{i}^{j}r_{i}^{F}$$

(6)

3.2.2 Optimal Demand for Capital

For notation simplicity, let $j = \{F, B\}$. I define the following notations:

$$\Gamma(\omega_{i,t+1}^{j}) = \int_{\omega_{i,t+1}^{j}}^{\omega_{i,t+1}} [\omega_{i,t+1}f(\omega_{i,t+1})d\omega_{i,t+1} + [1 - F(\omega_{i,t+1}^{j})]W_{i,t+1}^{j}]$$

(7)

\(^{15}\)In period $t+1$, aggregate risk associated with period-$t$ capital investment has been resolved. Therefore, expectation is taken solely over the remaining idiosyncratic risk.

\(^{16}\)Appendix A shows that this restriction holds under two assumptions, both of which are satisfied under a reasonable parameterisation.
Using equations (3), and (6)-(8), the optimal zero expected profit condition for depositor \(m\), is given by;

\[
(1 - \theta_s) \left[ \Gamma(W^P_{i,t+1}) - \theta_s D(W^P_{i,t+1}) \right] R_k - D_f^t = 0
\]

Banks are risk neutral and therefore are willing to bear both aggregate and idiosyncratic risks. Given that banks operate under a perfectly competitive environment, optimality conditions require that their expected profit functions conditional on both aggregate and idiosyncratic risks be equal to zero. Thus, using equations (1), (3), (5), (7), and (8), the optimal zero profit condition for bank \(j\) is given by;

\[
E_t \left[ \left[ \Gamma(W^P_{i,t+1}) - (1 - \theta_s) \Gamma(W^P_{i,t+1}) - \theta_s D(W^P_{i,t+1}) \right] R_k K_k - A_f^t \right] = 0
\]

where \(E_t(\cdot)\) denotes the real expectation conditional on information available in period \(t\).

As firm \(i\) is risk neutral, it maximises its expected profit function, where the expectation is conditional on both idiosyncratic and aggregate risks, subject to bank \(j\)’s balance sheet identity (equation (2)) and the zero expected profit conditions of depositor \(m\) and bank \(j\) (equations (9) and (10), respectively). \(^17\) I define \(k_t^j \equiv \frac{Q_t K_t^j}{(W_t^j + A_t^j)}\), \(s_t \equiv E_t \left( \frac{R_{k,t+1}^j}{r_{t+1}^j} \right)\), \(u_{t+1} \equiv \frac{R_{k,t+1}^j}{E_t(r_{t+1}^j)}\), \(e_t(r_{t+1}^j)\) where \(u_{t+1}\) captures the source of aggregate risk in the model, the firm’s optimisation problem taken as given the values of \(W_t^i, A_t^i, E_t(R_{k,t+1}^j), E_t(r_{t+1}^j)\) and \(Q_t\) can be written as follows;

\[
\max_{k_t^j, s_t, u_{t+1}} E_t \left( \sum_{j=0}^{\infty} (1 - \Gamma(W^P_{i,t+1}))[u_{t+1+j} s_{t+j} k_t^i] \right) \text{ subject to } \]

\[
(1 - \theta_s) \left[ \Gamma(W^P_{i,t+1}) - \theta_s D(W^P_{i,t+1}) \right] u_{t+1}s_t - (k_t^i - 1) = 0
\]

\[
E_t \left[ \Gamma(W^P_{i,t+1}) - (1 - \theta_s) \Gamma(W^P_{i,t+1}) - \theta_s D(W^P_{i,t+1}) \right] u_{t+1}s_t - A_f^t / \left( W_t^i + A_t^i \right) = 0
\]

The solution to the maximisation problem is given in Appendix B. Given that \(k_t^i > 1\), the first order

\(^{17}\) Intuitively, the solution to the maximisation problem yields firm \(i\)’s optimal demand for capital. Given the firm’s net worth, this directly implies a schedule of demand for loans. The expected zero profit condition for depositor \(m\) then implies a schedule of supply of deposits. Under a perfectly competitive environment, the equilibrium spread between the lending and deposit rates is determined so as banks yield a zero expected profit. Then the equilibrium level of the two rates are chosen so as the bank’s balance sheet identity holds. Thus the model crucially assumes that the lending and deposit rates always perfectly and costlessly adjust to their respective equilibrium rates. Although it greatly simplifies the analysis, it is worth mentioning that the assumption is rarely satisfied in reality owing the the prevalence of the information cost. Banks cannot perfectly forecast the ex post demand for loans by firms and the supply of deposits from depositors when they ex ante announce their lending and deposit rates. Thus, the ex post demand for loans may not equal to the sum of the ex post supply of deposits and bank capital. However, bank balance sheets have to be balanced at any point in time. This gives rise to the role of ‘short term government bond’ as a buffer stock.

\(^{18}\) Otherwise the sum of the firm’s and bank’s inside capital would be sufficient to finance the firm’s investment outlay, in which case the bank does not need to obtain any deposits from the depositor. To illustrate, when \(W_t^i / (W_t^j + A_t^j) < k_t^i \leq 1\), entrepreneur \(i\)’s inside capital is insufficient to finance his investment project. He therefore has to borrow from bank \(j\).
necessary conditions from the maximisation problem yield the following optimal demand for capital:

\[ k_i^t = \Psi_t(s_t, A_i^t/(W_i^t + A_i^t)), \text{where } \frac{\partial \Psi_t(s_t, A_i^t/(W_i^t + A_i^t))}{\partial s_t} > 0, \frac{\partial \Psi_t(s_t, A_i^t/(W_i^t + A_i^t))}{\partial A_i^t/(W_i^t + A_i^t)} < 0 \]  

(11)

Equation (11) describes the key relationship in the model as it crucially implies that the Modigliani-Miller (1958) theorem does not hold. In particular, it effectively relates the financial positions of the agents to the real capital investment decision of the firm. The rationale underlying the strictly positive sign of the first derivative is as follows. Other things constant, a higher expected discounted return on capital investment \( s_t \) decreases the expected default probability of the firm and the bank. This attracts more savings from the depositor and allows the entrepreneur to expand the size of the firm.

The second derivative is strictly negative because, other things constant, a higher \( A_i^t/(W_i^t + A_i^t) \) implies that the agency problem is relatively more severe at the firm level, as opposed to the bank level. The bank in turn has to impose a higher interest rate margin between the non-default lending and deposit rates. As the firm faces a relatively higher cost of borrowing, its capital demand declines. However, under a reasonable parameterisation, the magnitude of the latter derivative is so small that ignoring the effect of changes in \( A_i^t/(W_i^t + A_i^t) \) on the firm’s optimal demand for capital does not affect the dynamics of the model.\(^{19}\) Thus, equation (11) can be approximately written as:

\[ k_i^t \approx \psi(s_t), \psi'(s_t) > 0 \]  

(12)

Alternatively equation (13) can be rearranged to express the following inverse demand for capital:

\[ s_t = \psi^{-1} (k_i^t), \frac{\partial \psi^{-1}(s_t)}{\partial k_i^t} > 0 \]  

(13)

Equation (18) implies that as firm \( i \) and bank \( j \) increase their leverage, or equivalently their financial positions worsen, the expected discounted return to capital—which can also be interpreted as the external finance premium—has to increase. The key to understand this relationship is to recognise the link between the agents’ financial positions and the market interest rates which is captured by equations (1), (3), (9) and (10). Other things constant, a higher demand for capital relative to the sum of the firm’s and bank’s inside capital \( (\uparrow k_i^t) \) implies that depositor \( m \) is exposed to a higher agency problem.

This implies, via equations (3) and (9), that a higher non-default deposit rate is required \((\uparrow r^D_{i,t+1})\) in

\footnote{However, bank \( j \)'s inside capital alone is sufficient to finance the demand for loans by entrepreneur \( i \). Thus the source of external finance premium in this case draws solely from financial friction at the firm level. Essentially, this case is consistent with Bernanke et al. (1999). If \( k_i^t < W_i^t/(W_i^t + A_i^t) \), entrepreneur \( i \)'s inside capital is sufficient to finance his own investment outlay. In this case, there will be no external finance premium, \( s_t = 1 \). This is the standard case for models in which financial friction is absent.

\(^{19}\)By taking into account the effect of \( A_i^t/(W_i^t + A_i^t) \) on \( k_i^t \) in the simulation analysis (not reported), the result in terms of the dynamic responses of the key variables is virtually the same as the case when the effect is ignored.}
order to induce him to supply more savings. Given rational expectation, bank \( j \) anticipates a higher borrowing cost, via equations (1) and (10), the non-default loan rate has to increase (\( \uparrow r_{t,t}^{L} \)) in order to satisfy the bank’s zero expected profit condition. This directly imposes a greater cost of borrowing on firm \( i \) which in turn implies that a higher external finance premium (\( \uparrow s_{t} \)) is required. Thus, it is the relationship \( \uparrow k_{t} \Rightarrow \uparrow r_{t,t}^{D} \Rightarrow \uparrow r_{t,t}^{L} \Rightarrow \uparrow s_{t} \)’ that underpins the positive sign of the derivative shown in equation (13).

### 3.3 The Evolution of Entrepreneurial Net Worth and Bank Capital

Thus far, entrepreneurial net worth (\( W_{i,t} \)) and bank capital (\( A_{i,t} \)) have been taken as given. This subsection endogenises these two variables.

As a technical matter, it is necessary to start entrepreneurs off with some net worth in order to allow them to begin operation. I assume that, in each period, each of them is endowed with a small endowment, \( e^{F} \). In order to prevent them from accumulating sufficient wealth to become self-financed, I assume that they face a constant probability of dying, \( \gamma^{E} \). A dying entrepreneur simply consumes his remaining net worth (\( C_{E,i,t} \)) and departs from the scene. Given these specifications, we can write the evolution of firm \( i \)’s expected net worth and its consumption as:

\[
W_{i,t} = (1 - \gamma^{E}) \left[ (1 - \Gamma(\overline{F}_{i,t})) Q_{t-1} R_{t}^{K} K_{t-1} + e^{F} \right] \\
C_{E,i,t} = (\frac{\gamma^{E}}{1 - \gamma^{E}}) W_{i,t} 
\]

(14)\hspace{1cm}(15)

where \( W_{i,t} \) is firm \( i \)’s expected net worth available right before period-\( t \) capital decision is made. \( [1 - \Gamma(\overline{F}_{i,t})] Q_{t-1} R_{t}^{K} K_{t-1} \) is the expected entrepreneurial wealth accumulated from capital investment. The expectation is taken solely on the un realised idiosyncratic risk associated with the last period capital investment.

Similar to the case of firms, each bank is given a small endowment equal to \( e^{B} \) and faces a constant probability of dying, \( \gamma^{B} \). A dying bank simply consumes all of their remaining capital (\( C_{B,i,t} \)) and departs from the scene. The evolution of bank \( j \)’s capital and its consumption can be written as:

\[
A_{i,t} = (1 - \gamma^{B}) \left[ (1 - \theta^{B}) \Gamma(\overline{F}_{i,t}) - \theta^{B} G(\overline{F}_{i,t}) \right] R_{t}^{K} Q_{t-1} K_{t-1} + e^{B} \] \\
C_{B,i,t} = \frac{\gamma^{B}}{1 - \gamma^{B}} A_{i,t} 
\]

(16)\hspace{1cm}(17)

\[^{20}\] I assume that the cost of raising bank capital directly is prohibitively expensive. Thus bank capital can only be accumulated via retained earnings.

13
where $A_i^t$ is bank $j$’s expected bank capital in period $t$.\footnote{The expectation is taken conditional solely on the unrealised idiosyncratic risk associated with the capital investment in the previous period.} $\Gamma(\pi^F_{i,t}) - (1 - \theta^B)\Gamma(\pi^B_{i,t}) - \theta^B G(\pi^F_{i,t})] R^K_t Q_{t-1} K^t_{i-1}$ is the bank’s expected wealth accumulated from providing intermediary services.

## 3.4 Aggregation

In general, when demand for capital depends on the financial positions of agents, aggregation becomes difficult as it depends on the distribution of wealth amongst firms (similarly for banks). However, owing to the assumption of constant returns to scale throughout the paper, a firm’s demand for capital is proportional to its net worth with the factor of proportionality being the same for all firms (Bernanke et al, 1999). Similarly, each bank will optimally choose its lending in the same proportion to its inside capital. Given these results, the aggregation of a representative entrepreneur’s demand for capital, equation (13), becomes straightforward. Furthermore, the zero expected profit conditions for a representative depositor and a representative bank, given in equations (9) and (10), respectively, hold in aggregate. As a result, via equations (1) and (3), the non-default lending rate (non-default deposit rate) charged to different firms (banks) will be the same.\footnote{The intuition is as follows. Since all firms have the same leverage ratio, they possess the same degree of risk \emph{ex ante}. This implies that banks will charge the same non-default lending rate to all firms. Similarly, as all banks have the same leverage ratio, depositors are exposed to the same degree of risk \emph{ex ante}. As a compensation, they would thus universally charge the same non-default deposit rate to all banks.}

In all, equations (1), (3), (9)-(10), (13)-(17) hold in aggregate, i.e. without the super/subscript $i$. They represent the key non-standard building block of the model. In particular, equations (1), (3), (9), (10) effectively links entrepreneurial net worth and bank capital to the equilibrium non-default lending and deposit rates. These links underpin the mechanism by which the financial positions of firms and banks work to augment the real investment decision of firms. This mechanism, which is completely absent in the frictionless models, is captured by equation (13). Equations (14) and (16) then characterise the evolution of entrepreneurial net worth and bank capital, respectively. Lastly, equations (15) and (17) determine the consumption of entrepreneurs and banks, respectively.
4 Model Simulation

To quantitatively show how the dynamic evolution of bank capital operates to enrich the transmission mechanism of monetary policy by augmenting the dynamic of aggregate macro variables, e.g. investment, output and asset price, I embed the 9 key equations obtained from the previous section (equations (1), (3), (9)-(10), (13)-(17)) into the otherwise standard DNK model. As much of the general equilibrium specification of the model is by now standard, to conserve space, this section abstracts from deriving the standard equations, and reports only the simulation results obtained from log-linearising the full model around a unique steady state equilibrium.  

4.1 Calibration

The model is calibrated at a quarterly frequency. The values assigned to most of the parameters relevant to preference, technology and price stickiness are standard in the DNK literature. The discount factor is set to be 0.99. The depreciation rate is set to 2.5 percent. I select the steady state capital share to be 0.35, the labour supply elasticity to be 3 and, following Bernanke et al. (1999), the elasticity of the price of capital with respect to the investment capital ratio to be 0.25. The steady state mark-up price is set to 1.05. The probability that a retail firm does not change its price in a given period is chosen to be 0.75, implying an average price duration of one year. The parameters in the policy rule for lagged inflation and lagged risk free rate are set to 0.11 and 0.9, respectively.

The unconventional choices of parameterisation are those relevant to the financial contract problem. The endowment given to each firm and bank as a proportion of their inside capital, \( \frac{e^E}{W} \) and \( \frac{e^B}{A} \), is set to be 0.01. I then treat the proportional factors of the verification cost paid by a bank \( (\theta^B) \), and a depositor \( (\theta^D) \), the death rate for a firm \( (\gamma^E) \) and bank \( (\gamma^B) \), and the standard deviation of \( \ln \pi \) \( (\sigma) \) as unobservable and choose their values to match the following steady state outcomes: [1] an annualised risk spread, \( R^k - r^f \), of 200 basis points, approximately the historical average spread between the prime

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23The derivation of the standard equations can be found, amongst others, in the working paper version of this paper, Sunirand (2002), or Appendix B of Bernanke et al. (1999). Briefly, entrepreneurs purchase capital one period in advance, and hire households’ labour to produce wholesale goods according to a constant returns to scale production function. Households are infinitely-lived agents who consume, save (by depositing money with banks), and supply labour. Retailers purchase wholesale goods from entrepreneurs in the competitive market, differentiate it without resource costs, and resell it as retail goods. The introduction of the retail sector is meant to introduce a Calvo’s (1983) type of price stickiness without complicating the aggregation process. The CES aggregates of retail goods are then used for households’ consumption, entrepreneurs’ investment, and an aggregate verification cost. The central bank conducts monetary policy by setting the (nominal) risk free rate according to a Taylor-type rule.

24Its magnitude is meant to be so small that it does not affect the model’s dynamics.
lending rate and the six-month Treasury bill rate; [2] an annualised business failure rate, \( F(\bar{F}) \), of 3 percent, the approximate value in the data (Bernanke et al., 1999); [3] an annualised bank failure rate, \( F(\bar{B}) \), of 0.5 percent, the average value for commercial banks insured by Federal Deposit Insurance Corporation (FDIC) during 1988-2002\(^{25}\); [4] a leverage ratio of entrepreneurial net worth to capital (\( \frac{W}{K} \)) of 0.4, the approximate value found in the empirical literature for small firms in the U.S.\(^{26}\); [5] a bank’s capital to asset ratio, \( \frac{A}{L} \), of 0.12, the average value of \( (\text{tier 1 capital} + \text{tier 2 capital}) \) risk-weighted asset for U.S. commercial banks during 1990-1999\(^{27}\). To satisfy [1]-[5], the values of \( \theta_B, \theta_D, \gamma_E, \gamma_B \) and \( \sigma \) are 0.01, 0.4, 0.03, 0.02, and 0.2, respectively.

### 4.2 The Transmission Mechanism of Monetary Policy: The Role of Bank Capital

Figure 1 shows the response of the model to an unanticipated rise in the nominal risk-free rate by 1 percent from the steady state. Owing to price stickiness, the real risk free rate rises correspondingly. Consumption and thus aggregate demand fall via the standard intertemporal substitution effect. Because capital stock has to be purchased one period in advance, an unexpected decline in aggregate demand causes the return to capital purchased in the previous period to fall. As depositors are completely hedged against any realisation of aggregate risk, the non-default deposit rate associated with deposit contracts signed in the previous period rises instantaneously to compensate them for the lower-than-expected realisation of return to capital, \( R^K_t < E_{t-1}(R^K_t) \), as well as the higher-than-expected realisation of their opportunity cost of funds, \( r^f_t > E_{t-1}(r^f_t) \). This directly imposes a higher cost of borrowing on banks. However, the lending rate associated with loan contracts signed in the previous period is predetermined as of period \( t \). This in turn implies that banks’ interest rate cost has to rise relatively faster compared to their interest rate revenue, thereby depleting banks’ inside capital. Moreover, a lower than expected return to capital decreases entrepreneurial net worth directly. The decline in both firms’ and banks’ inside capital means that both firms and banks have less to contribute to firms’ investment project which in turn implies that depositors are exposed to a higher agency cost.

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\(^{25}\)Source: Quarterly Banking Profile, FDIC.

\(^{26}\)Using data from the 1993 National Survey of Small Business Finances, Gibson (2002) found that equity accounts for approximately 40 percent of U.S. small firms’ overall source of finance. It should be noted that the emphasis is placed on small firms, as opposed to large firms, because they are relatively more bank dependent which can be more appropriately rationalised in this model where bank loans are the only source of external finance.

\(^{27}\)Source: OECD, Bank Profitability.
As a compensation, the non-default deposit rate associated with deposit contracts signed in period $t$ (which will not be realised until period $t+1$) has to rise. Rational expectation then implies that banks will have to increase their non-default lending rate associated with period-$t$ contract immediately, in anticipation of the higher expected future interest rate cost. Given an increase in the non-default lending rate, firms face a higher cost of borrowing, implying that the external finance premium, $s_t$, must rise. Demand for capital, investment and asset price have to fall as the firm’s external cost of borrowing becomes more expensive. A kind of multiplier effect then arises as a higher non-default lending rate, together with a lower asset price, decrease entrepreneurial net worth further in the subsequent period. Moreover, in response to the initial fall, bank capital slowly accumulates back to trend at the rate equivalent to the real risk free rate. Given that the accumulation process is slow enough, bank capital will be persistently below its trend. The negative effect of the persistent decline in bank capital and entrepreneurial net worth then feeds into the subsequent periods by aggravating the deposit rate, the lending rate, and therefore the external finance premium, which in turn works to depress the demand for capital, investment and aggregate output further.

As a theoretical counterpart to the results shown in Figure 1, Figure 2 summarises how the transmission mechanism of monetary policy works in the model. Crucially, it exhibits the unconventional transmission channel in which monetary policy affects aggregate macro variables via its effect on bank capital; the bank capital channel. This channel and its dynamic interplay with the balance sheet channel (via entrepreneurial net worth), and asset price channel (via price of capital) work to ‘augment’ the investment decision of firms by exacerbating the two-sided agency problem. I turn now to the issue of the amplification and propagation properties of the model.

4.3 Amplification and Propagation Mechanisms

The importance of the role of bank capital in amplifying as well as propagating the response of aggregate real economic activities to a monetary shock can be seen from Figure 3. In the figure, I compare the dynamic responses of the model to a negative monetary shock with those of the frictionless model (FLM) and the model with bank capital channel turned off (NBM). In the FLM, the roles of entrepreneurial net worth and bank capital are completely shut off as firms can borrow external funds at the sole opportunity cost equivalent to the risk free rate. For the NBM, I ignore the assumption that depositors
are averse to aggregate risk but assume instead that they are risk neutral. As a result, similar to banks, depositors are willing to bear aggregate risk. This eliminates the operational mechanism of the bank capital channel discussed earlier, as the adjustment of the lending rate to aggregate shock is no longer stickier compared to that of the deposit rate. This implies that, in response to a negative monetary shock, the interest rate cost may not rise faster compared to its revenue counterpart and thus bank capital may not decline. Put differently, \textit{turning off the bank capital channel is analogous to assuming away the effect of having a maturity mismatch in banks’ balance sheet.}

As can be seen from the figure, incorporating the bank capital channel into the model amplifies as well as propagates the effect of a negative monetary shock on aggregate output and investment. The initial response of investment is nearly twice as great compared to that of FLM and approximately 25 percent greater compared to that of NBM. As for output, the effect is approximately 65 percent greater compared to that of FLM and 20 percent greater compared to that of NBM. Moreover, the persistence of the real effects implied by the model is significantly greater compared to those of FLM and NBM. Evidently, the responses of investment and output persist the least under FLM. This is because the roles of entrepreneurial net worth, bank capital and therefore the external finance premium are passive under FLM. More interestingly, when the bank capital channel is turned on, in comparison to the NBM, the persistence of the real effect is evidently larger; e.g. relative to trend, investment and output in the full model after 10 quarters are about where they are in the NBM after 6 quarters. Although the responses of entrepreneurial net worth are not markedly different under the two models, the responses of bank capital are indeed very different. When the bank capital channel is shut off, the immediate response of bank capital declines slightly and turns positive in the subsequent periods. Thus, unlike the role of bank capital in the full model, the response of bank capital in the NBM operates to \textit{lessen} the overall agency problem. The relatively more active role of bank capital in \textit{magnifying} the agency problem in the full model is mirrored by a substantially stronger response of the external finance premium.

All in all, in the terminology of Bernanke et. al (1999), the simulation result shows that the transmission mechanism of monetary policy implied by the model exhibits a \textit{financial accelerator effect} in that endogenous evolution of bank capital, together with that of entrepreneurial net worth and asset price, work to amplify as well as propagate the effect of monetary shocks in the macroeconomy.
5 Conclusion

In this paper, I propose a model to illuminate how the monetary policy transmission process from its initial impulse to the ultimate response on aggregate economic activities could get amplified and propagated through its effect on bank capital; the bank capital channel. This channel and its dynamic interplay with the balance sheet channel and the asset price channel work to augment the real investment decision of firms by magnifying the two-sided agency problem, the double Costly State Verification. The simulation results confirm the quantitative importance of the financial accelerator effect in that endogenous evolution of bank capital operates to amplify and propagate the effect of a monetary shock in the macroeconomy.

To keep the model as simple as possible, the banking sector in this model has been highly simplified. Amongst others, it abstracts from the fact that banks in reality hold other assets besides loans, and acquire external funds from other sources besides short-term deposits. Relaxing these simplifying assumptions would allow us to identify other transmission channels in which monetary policy affects the real macroeconomy via the banking system. This task is left for future research.

6 Appendix A

This appendix shows the assumptions which ensure that $\overline{F}_{i,t+1}$ is strictly greater than $\overline{B}_{i,t+1}$ in equilibrium. In general, 3 scenarios are plausible concerning the relative values of $\overline{F}_{i,t+1}$ and $\overline{B}_{i,t+1}$. For notational simplicity, I ignore the time subscript in this appendix.

**Scenario 1:** $\overline{F}_{i} \leq (1 - \theta^B)\overline{B}_{i}$: Under this scenario, equations (1) and (3) in the text imply that $r_{L}^{i}L^{i} \leq r_{D}^{i}D^{i}$. Given a strictly positive opportunity cost for bank $j$, $A^{i}r^{j} > 0$, the bank will always go bankrupt as its revenue from lending can never cover its cost. Thus we can dismiss this scenario as a potential equilibrium solution.

**Scenario 2:** $(1 - \theta^B)\overline{F}_{i} < \overline{F}_{i} \leq \overline{B}_{i}$: When $\overline{w}_{i} < \overline{F}_{i}$, the firm will go bankrupt. After paying the verification cost, bank $j$ receives $(1 - \theta^B)\omega_{i}QRK^{i}$ as its net liquidation revenue. Since this revenue is less than the bank’s obligation to repay depositor $m$, i.e. $(1 - \theta^B)\omega_{i}QRK^{i} < r_{D}^{i}D^{i}$, the bank will go bankrupt.

When $\overline{w}_{i} > \overline{F}_{i}$, firm $i$ is able to pay the bank according to the contract, $r_{L}^{i}L^{i}$. Since the bank’s
revenue in this case is enough to fulfil the deposit contract, \( r_i^L L^i = \overline{\omega}_i^F QK K^i > (1 - \theta^B)\overline{\omega}_i^B QK K^i = r_i^D D^i \), the bank does not default and pockets \( r_i^L L^i - r_i^D D^i \). Given the opportunity cost of funds equivalent to the real risk-free rate, \( r^f \), using equations (1) and (3) in the text, the bank’s expected profit function conditional on the realisation of idiosyncratic risk is given by;

\[
\pi^B_{i(1-\theta^B)\overline{\omega}_i^B \leq \overline{\omega}_i^F} = \left( \overline{\omega}_i^F - (1 - \theta^B)\overline{\omega}_i^B \right)[1 - F(\overline{\omega}_i^F)]QK K^i - A^i r^f \tag{A1}
\]

**Scenario 3:** \( \overline{\omega}_i^F > \overline{\omega}_i^B \): When \( \overline{\omega}_i < \overline{\omega}_i^B \), firm \( i \) goes bankrupt. Since bank \( j \)’s revenue after paying the verification cost is insufficient to fulfil the deposit contract, i.e. \( (1 - \theta^B)\overline{\omega}_i^B R^K K^i < (1 - \theta^B)\overline{\omega}_i^F R^K K^i = r_i^D D^i \), it declares a default. When \( \overline{\omega}_i^F > \overline{\omega}_i \geq \overline{\omega}_i^B \), the firm remains bankrupt. However, the bank’s revenue netting off the verification cost is now enough to fulfilling the deposit contract, \( (1 - \theta^B)\overline{\omega}_i^F R^K K^i \geq (1 - \theta^B)\overline{\omega}_i^B R^K K^i = r_i^D D^i \). Hence the bank pockets \( (1 - \theta^B)\overline{\omega}_i^F R^K K^i - r_i^D D^i \). Lastly, when \( \overline{\omega}_i \geq \overline{\omega}_i^F \), both the bank and the firm do not declare bankruptcy. The bank would then receive \( r_i^L L^i - r_i^D D^i \) as its profit.

The bank’s expected profit conditional on the realisation of \( \overline{\omega}_i \) in this case is given by;

\[
\pi^B_{\overline{\omega}_i^F > \overline{\omega}_i^B} = \int_{\overline{\omega}_i^F}^{\overline{\omega}_i^B} [(1 - \theta^B)\overline{\omega}_i^F - r_i^D D^i/f(\overline{\omega}_i^F)]d\overline{\omega}_i + [1 - F(\overline{\omega}_i^F)] [r_i^L L^i - r_i^D D^i] - A^i r^f \tag{A2}
\]

Using the simplifying notations given in the text (equations (7)-(8)) together with equations (1) and (3), equation (A2) can then be rewritten as;

\[
\pi^B_{\overline{\omega}_i^F > \overline{\omega}_i^B} = \left( \Gamma(\overline{\omega}_i^F) - (1 - \theta^B)\Gamma(\overline{\omega}_i^B) - \theta^B G(\overline{\omega}_i^F) \right) R^K K^i - A^i r^f \tag{A3}
\]

For notational simplicity, define \( R^B \equiv \frac{\pi^B_{\overline{\omega}_i^F > \overline{\omega}_i^B}}{\Gamma(\overline{\omega}_i^B)} \). From equations (A1) and (A3), I can write,

\[
R^B = \begin{cases} (\overline{\omega}_i^F - (1 - \theta^B)\overline{\omega}_i^B)[1 - F(\overline{\omega}_i^F)] & \text{scenario 2} \\ \left[ \Gamma(\overline{\omega}_i^F) - (1 - \theta^B)\Gamma(\overline{\omega}_i^B) - \theta^B G(\overline{\omega}_i^F) \right] & \text{scenario 3} \end{cases}
\]

Firstly, take limit of \( R^B \) at \( \overline{\omega}_i^F = \overline{\omega}_i^B \):

\[
\lim_{\overline{\omega}_i^F \to \overline{\omega}_i^B} R^B = \theta^B \overline{\omega}_i^B [1 - F(\overline{\omega}_i^B)] \geq 0 \tag{A4}
\]

Next, I take the partial derivative of \( R^B \) with respect to the threshold value \( \overline{\omega}_i^F \) for both scenarios 2 and 3, and then take limit at \( \overline{\omega}_i^F = \overline{\omega}_i^B \) to obtain;

\[
\lim_{\overline{\omega}_i^F \to \overline{\omega}_i^B} \frac{dR^B}{d\overline{\omega}_i^F} = [1 - F(\overline{\omega}_i^B)] [1 - \theta^B \overline{\omega}_i^B h(\overline{\omega}_i^B)] \tag{A5}
\]

where, as defined in the text, \( h(\overline{\omega}_i) \equiv \frac{f(\overline{\omega}_i)}{1 - F(\overline{\omega}_i)} \) is the hazard rate.

As \( \overline{\omega}_i \) is log normally distributed, it satisfies the increasing hazard rate restriction (see footnote 5). This implies that \( R^B \) reaches a global maximum at a unique \( \overline{\omega}_i^F \) and is an increasing function for \( \overline{\omega}_i^F < \overline{\omega}_i^B \).
Since \( \overline{\omega}_i^e > \overline{\omega}_i \) can never be an equilibrium, it must be that \( \frac{\partial R^B}{\partial \overline{\omega}_i} > 0 \) in order to ensure a non-rationing equilibrium outcome. Thus a necessary condition to ensure \( \overline{\omega}_i^e > \overline{\omega}_i^B \) is that \( \overline{\omega}_i^B \) be less than \( \overline{\omega}_i^e \). To achieve this, from equation (A5), the following assumption must hold;

**Assumption A1:** \( [1 - F(\overline{\omega}_i^B)][1 - \theta^B h(\overline{\omega}_i^B)] > 0 \)

From equation (A4), another assumption to ensure that equilibrium \( \overline{\omega}_i^e \) will be greater than \( \overline{\omega}_i^B \) is given as follows;

**Assumption A2:** \( \theta^B \overline{\omega}_i^B [1 - F(\overline{\omega}_i^B)] < \frac{\mu_i^e}{\kappa K} \)

Assumption A2 implies that \( \overline{\omega}_i^e = \overline{\omega}_i^B \) cannot be the equilibrium as the opportunity cost of funds outweighs the expected revenue. Assumptions A1 and A2 together then imply that equilibrium \( \overline{\omega}_i^e \) must lie within the range \( (\overline{\omega}_i^B, \overline{\omega}_i^e) \) given that bank’s opportunity cost is not too high, i.e. \( \frac{\mu_i^e}{\kappa K} \leq R^B |_{\overline{\omega}_i^e = \overline{\omega}_i^B} \). If bank’s opportunity cost is too high, \( \frac{\mu_i^e}{\kappa K} > R^B |_{\overline{\omega}_i^e = \overline{\omega}_i^B} \), the firm is rationed.

### 7 Appendix B

In this appendix, I derive the optimality conditions for the firm’s demand for capital. For notational simplicity, I drop the \( i \) subscript. To begin, I rewrite equations (1) and (3) in the text as follows;

\[
\begin{align*}
\overline{\omega}_{t+1}^e &= \frac{r_{t+1}^e (k_t - [W_t/(W_t + A_t)])}{r_{t+1} (k_t - 1)} \\
\overline{\omega}_{t+1}^B &= \frac{r_{t+1}^B (k_t - 1)}{(1 - \theta^B) r_{t+1} (k_t - 1)}
\end{align*}
\]  

(B1)  

(B2)

The first order necessary conditions from the firm’s optimisation problem stated in the text are as follows;

\[
\begin{align*}
\frac{\lambda_{t+1}^2}{\lambda_t^1} &= \frac{\Gamma'(\overline{\omega}_{t+1}^B)}{\Gamma'(\overline{\omega}_{t+1}^e) - \theta^D G'(\overline{\omega}_{t+1}^e)} \\
\frac{\lambda_t^1}{\lambda_t^1} &= \frac{E_t [\Gamma'(\overline{\omega}_{t+1}^e) \frac{\partial \overline{\omega}_{t+1}^e}{\partial u_{t+1}} u_{t+1}]}{E_t [\Gamma'(\overline{\omega}_{t+1}^e) - G'(\overline{\omega}_{t+1}^e)] \frac{\partial \overline{\omega}_{t+1}^e}{\partial u_{t+1}} u_{t+1}]
\end{align*}
\]  

(B3)  

(B4)

where \( \lambda_t^1 \) is the \textit{ex-ante} value of the Lagrange multiplier on the constraint that bank \( j \) earns zero expected profit \textit{prior} to the realisation of aggregate risk and \( \lambda_{t+1}^2 \) is the \textit{ex-post} value of the Lagrange multiplier on the constraint that depositor \( m \) earns zero expected profit \textit{after} the realisation of aggregate risk.

\[
\begin{align*}
k_t : J_{J_t}(s_t, \lambda_t^1, \lambda_t^2) &= E_t [\varphi_{t+1} u_{t+1} s_t - \lambda_{t+1}^2] = 0 \\
\varphi_{t+1} &= [1 - \Gamma(\overline{\omega}_{t+1}^e)] + \lambda_{t+1}^2 \Gamma(\overline{\omega}_{t+1}^e) - (1 - \theta^B) \Gamma(\overline{\omega}_{t+1}^e) - \theta^D G(\overline{\omega}_{t+1}^e) + \lambda_{t+1}^2 (1 - \theta^B) [\Gamma(\overline{\omega}_{t+1}^e) - \theta^D G(\overline{\omega}_{t+1}^e)] \\
\lambda_{t+1}^2 : (1 - \theta^B) [\Gamma(\overline{\omega}_{t+1}^e) - \theta^D G(\overline{\omega}_{t+1}^e)] u_{t+1} s_t (k_t - (k_t - 1)) &= 0
\end{align*}
\]  

(B5)  

(B6)
\( \lambda_1^1 : E_t \{ [\Gamma(\overline{\omega}_t^{F+1}) - (1 - \theta^B)\Gamma'(\overline{\omega}_t^{F+1})] - \theta^B G(\overline{\omega}_t^{F+1}) \} u_{t+1} + k_t - A_t/(W_t + A_t) = 0 \) \hspace{1cm} (B7)

From equations (B1)-(B7), there are 7 equations in 7 variables (\( \overline{\omega}_t^{F+1}, \overline{\omega}_t^{B+1}, r_{t+1}^f, r_{t+1}^b, k_t, \lambda_{t+1}^2, \lambda_{t+1}^1 \)) taken as given the values of \( s_t \) and \( \frac{A_t}{W_t + A_t} \). Thus, in equilibrium, it is possible to write \( k_t \) solely as a function of \( s_t \) and \( \frac{A_t}{W_t + A_t} \); \( k_t = \Psi(s_t, \frac{A_t}{W_t + A_t}) \). In what follows, I will show that \( \frac{\partial \Psi(s_t)}{\partial s_t} > 0 \) and \( \frac{\partial \Psi(s_t)}{\partial A_t} < 0 \).

From equations (B1) and (B2), given that \( k_t > 1 \), I could find the following derivatives:

\[
\frac{\partial \overline{\omega}_t^{B+1}}{\partial r_{t+1}^f} = (k_t - [W_t/(W_t + A_t)]) \geq 0 \quad \text{and} \quad \frac{\partial \overline{\omega}_t^{B+1}}{\partial r_{t+1}^b} = (1 - \theta^B)(k_t - 1) \geq 0
\]

(B8)

(B9)

Substituting equation (B8) into equation (B4), the latter equation can be rewritten as:

\[
\lambda_1^1 = \frac{E_t^f \{[\Gamma(\overline{\omega}_t^{F+1})] - 1 \}}{E_t^f \{[\Gamma(\overline{\omega}_t^{F+1}) - G'(\overline{\omega}_t^{F+1})] \}}
\]

(B10)

Given the increasing hazard rate assumption and the result from Appendix A that \( \overline{\omega}_t^F > \overline{\omega}_t^B \), \( \Gamma'(\overline{\omega}_t^{F+1}) \) and \( \Gamma'(\overline{\omega}_t^{B+1}) - \theta^B G'(\overline{\omega}_t^{F+1}) \) are strictly positive. These imply, via equations (B3) and (B10), that \( \lambda_1^1 > 0 \) and \( \lambda_{t+1}^2 > 0 \).

From equation (B10), taking partial derivative with respect to \( \overline{\omega}_t^{F+1} \) and using the fact that \( \Gamma'(\overline{\omega}_t^{F+1}) = 1 - F(\overline{\omega}_t^{F+1}) \) and \( G'(\overline{\omega}_t^{F+1}) = \overline{\omega}_t^{F+1} f(\overline{\omega}_t^{F+1}) \), where \( F(\overline{\omega}_t^{F+1}) \) and \( f(\overline{\omega}_t^{F+1}) \) are c.d.f. and d.f. of \( \overline{\omega}_t^{F+1} \) respectively, I can then write \( \frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{F+1}} \) as follows:

\[
\frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{F+1}} = \frac{\theta^B [E_t^f \{1 - F(\overline{\omega}_t^{F+1})\} E_t^f \{\frac{\partial (\overline{\omega}_t^{F+1}/f(\overline{\omega}_t^{F+1}))}{\partial \overline{\omega}_t^{F+1}} \} + \frac{\partial (\overline{\omega}_t^{F+1}/f(\overline{\omega}_t^{F+1}))}{\partial \overline{\omega}_t^{F+1}}] [E_t^f \{1 - F(\overline{\omega}_t^{F+1})\} - \theta^B E_t^f \{\frac{\partial (\overline{\omega}_t^{F+1}/f(\overline{\omega}_t^{F+1}))}{\partial \overline{\omega}_t^{F+1}}\}]^2}
\]

In general, the value of \( \frac{\partial (\overline{\omega}_t^{F+1}/f(\overline{\omega}_t^{F+1}))}{\partial \overline{\omega}_t^{F+1}} \) could be either positive or negative. However, under a reasonable parameterisation, it will be strictly positive in the neighbourhood of the steady state, in which case \( \frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{F+1}} > 0 \).

From equation (B3), take the partial derivative with respect to \( \overline{\omega}_t^{F+1} \) and \( \overline{\omega}_t^{B+1} \), I obtain:

\[
\frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{F+1}} = \frac{\Gamma'(\overline{\omega}_t^{F+1})}{\Gamma'(\overline{\omega}_t^{F+1}) - \theta^B G'(\overline{\omega}_t^{B+1})} \frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{F+1}} \geq \frac{\lambda_1^2}{\lambda_1^1} \frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{B+1}} > 0
\]

\[
\frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{B+1}} = \frac{\theta^B [\Gamma'(\overline{\omega}_t^{B+1}) - \theta^B G'(\overline{\omega}_t^{F+1})] \lambda_1^1}{\Gamma'(\overline{\omega}_t^{B+1}) - \theta^B G'(\overline{\omega}_t^{B+1})} \left[\left(\Gamma'(\overline{\omega}_t^{B+1}) - \theta^B G'(\overline{\omega}_t^{B+1})\right)\right]^2 \lambda_1^1
\]

Given the increasing hazard rate assumption, it follows directly that \( \frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{B+1}} > 0 \).

Thus far, I have established that \( \lambda_1^1, \lambda_{t+1}^2, \frac{\partial \lambda_1^1}{\partial \overline{\omega}_t^{F+1}}, \frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{F+1}}, \frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{B+1}}, \frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{B+1}}, \frac{\partial \lambda_1^2}{\partial \overline{\omega}_t^{B+1}} \) are all positive for \( \overline{\omega} \in (\overline{\omega}_t^B, \overline{\omega}_t^F) \). To show that \( \frac{\partial k_t}{\partial s_t} > 0 \), take the derivative of \( J_J(s_t, r_{t+1}^f, r_{t+1}^b) \) (equation (B5)) with respect to \( s_t \), and rearrange to obtain;
\[
\frac{\partial k_t}{\partial s_t} = -\frac{[\frac{\partial JJ_t}{\partial s_t} + \frac{\partial JJ_t}{\partial k_t} \frac{\partial r_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r_{t+1}} \frac{\partial r_t}{\partial k_t}]}{[\frac{\partial JJ_t}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}} \frac{\partial r_t}{\partial s_t}]} \tag{B11}
\]

From equation (B5),
\[
\frac{\partial JJ_t}{\partial s_t} = E_t(J_{t+1} s_{t+1} + 1) > 0
\]
\[
\frac{\partial JJ_t}{\partial r_t} = E_t[\Gamma^r(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1} + \lambda_1^r(\Gamma^r(\omega_{t+1}^E) - G^r(\omega_{t+1}^E)) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]
+ E_t[\frac{\partial JJ_t}{\partial r_t}((1 - \theta^B)\Gamma^r(\omega_{t+1}^E) - \theta^B G^r(\omega_{t+1}^E)) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]
+ E_t[\frac{\partial JJ_t}{\partial r_t}((1 - \theta^B)\Gamma^r(\omega_{t+1}^E) - \theta^B G^r(\omega_{t+1}^E)) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]
\]

Substituting equations (B3), (B4), (B6), and (B7) in equation (B12), after some algebraic manipulation, I obtain:
\[
\frac{\partial JJ_t}{\partial r_t} = E_t \left( \frac{\partial JJ_t}{\partial r_t} \left( \frac{1}{\lambda_1^r} \right) \left[ \frac{A_t}{W_t + A_t} \frac{1}{\lambda_1^r} \right] \right) \tag{B13}
\]
Because \(\frac{\partial JJ_t}{\partial r_t} > 0\), \(\frac{A_t}{W_t + A_t} \leq 1\), and \(\frac{1}{\lambda_1^r} < 1\), it follows that \(\frac{\partial JJ_t}{\partial r_t} < 0\).
\[
\frac{\partial JJ_t}{\partial r_{t+1}} = E_t[-\lambda_1^r(1 - \theta^B)\Gamma^r(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1} + \lambda_1^r(1 - \theta^B)\Gamma^r(\omega_{t+1}^E) - \theta^B G^r(\omega_{t+1}^E)] \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}
+ E_t[\frac{\partial JJ_t}{\partial r_t}((1 - \theta^B)\Gamma^r(\omega_{t+1}^E) - \theta^B G^r(\omega_{t+1}^E)) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]
\]

Using equations (B3) and (B6), we can write:
\[
\frac{\partial JJ_t}{\partial r_{t+1}} = -E_t[\frac{\partial JJ_t}{\partial r_t} \frac{1}{\lambda_1^r} \left( \frac{1}{\lambda_1^r} \right) < 0 \tag{B14}
\]
From equation (B6), I take the partial derivative with respect to \(s_t\) and \(k_t\), respectively, and rearrange to obtain:
\[
\frac{\partial r_{t+1}}{\partial s_t} = -\frac{\Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E)}{\Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}} < 0 \tag{B15}
\]
\[
\frac{\partial r_{t+1}}{\partial k_t} = \frac{(1 - \theta^B) \Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}}{\Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}} > 0 \tag{B16}
\]
From equation (B7), I take the partial derivative with respect to \(s_t\) and \(k_t\), respectively. Using equations (B6), B(7), (B15) and (B16), after some algebraic manipulation, I obtain:
\[
\frac{\partial r_t}{\partial s_t} = -\frac{E_t[\frac{\partial JJ_t}{\partial s_t} \frac{1}{\lambda_1^s} \left( \frac{1}{\lambda_1^s} \right) + \frac{A_t}{W_t + A_t} \frac{1}{\lambda_1^s}]}{E_t[\Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]} < 0 \tag{B17}
\]
\[
\frac{\partial r_t}{\partial k_t} = \frac{1}{\lambda_1^s} \frac{E_t[\frac{\partial JJ_t}{\partial s_t} \frac{1}{\lambda_1^s} \left( \frac{1}{\lambda_1^s} \right) + \frac{A_t}{W_t + A_t} \frac{1}{\lambda_1^s}]}{\Gamma(\omega_{t+1}^E) - \theta^B G(\omega_{t+1}^E) \frac{\partial r_{t+1}}{\partial r_t} s_{t+1}]} \tag{B18}
\]
Because, \(\frac{\partial JJ_t}{\partial s_t} > 1\) and \(\frac{A_t}{W_t + A_t} \leq 1\), it follows that \(\frac{\partial r_t}{\partial s_t} < 0\).

Thus far I have shown that \(\frac{\partial JJ_t}{\partial r_t} \frac{\partial JJ_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial s_t} \frac{\partial r_t}{\partial s_t} \frac{\partial r_t}{\partial k_t} \frac{\partial r_t}{\partial k_t} \), and \(\frac{\partial JJ_t}{\partial s_t} \frac{\partial JJ_t}{\partial s_t} \), and \(\frac{\partial JJ_t}{\partial k_t} \frac{\partial JJ_t}{\partial k_t} \) are strictly positive. Plugging these values into equation (B11) implies that \(\frac{\partial k_t}{\partial s_t}\) is strictly positive as required.
I turn now to show that $\frac{\partial k_t}{\partial [A_t/(W_t + A_t)]} < 0$. From equation (B5), take the derivative of $JJ_t$ with respect to $\frac{A_t}{W_t + A_t}$ and rearrange to obtain;

$$\frac{\partial k_t}{\partial [A_t/(W_t + A_t)]} = -\frac{[\frac{\partial JJ_t}{\partial r} \frac{\partial r L_t}{\partial k_t} + \frac{\partial JJ_t}{\partial r} \frac{\partial r D_t}{\partial k_t}]}{[\frac{\partial JJ_t}{\partial r} \frac{\partial r L_t}{\partial k_t} + \frac{\partial JJ_t}{\partial r} \frac{\partial r D_t}{\partial k_t}]}$$ (B19)

Now I take the derivative of equation (B7) with respect to $\frac{A_t}{W_t + A_t}$ and rearrange to obtain;

$$\frac{\partial r L_t}{\partial [A_t/(W_t + A_t)]} = \frac{1}{E_t \{\Gamma(\{\eta \alpha_{t+1}^\prime\} - \theta^B G(\{\eta \alpha_{t+1}^\prime\}) \frac{\partial \eta_{t+1}^\prime}{\partial k_t} u_{t+1}\} s_t k_t} > 0$$ (B20)

As $\frac{\partial JJ_t}{\partial r}$ and $\frac{\partial JJ_t}{\partial r} \frac{\partial r D_t}{\partial k_t}$ are strictly negative and $\frac{\partial r L_t}{\partial [A_t/(W_t + A_t)]}$, $\frac{\partial r L_t}{\partial k_t}$, and $\frac{\partial r D_t}{\partial k_t}$ are strictly positive, by substituting these values into equation (B19), it must be that $\frac{\partial k_t}{\partial [A_t/(W_t + A_t)]} < 0$.

In sum, I have shown in this appendix that $\frac{\partial k_t}{\partial r} > 0$, $\frac{\partial k_t}{\partial [A_t/(W_t + A_t)]} < 0$.
References


Figure 1: All Panels: Time horizon in quarters
End of Period $t$

Unanticipated rise in nominal interest rate

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Price stickiness

Higher real interest rate

Intertemporal substitution

Aggregate demand falls

Output falls

Inflation falls

Return to capital which was purchased last period falls

Deposit rate (with deposit contract signed last period) rises

Entrepreneurial net worth falls

Bank Capital falls

External finance premium rises

Demand for capital falls

Asset price falls

Investment falls

End of Period $t + 1$

Deposit rate (with the contract signed last period) rises

Lending rate (with loan contract this period) rises

Entrepreneurial net worth falls

Bank capital falls

External finance premium rises

Demand for capital falls

Investment falls

Asset price falls

Output falls

Expectation

Figure 2
Figure 3: All Panels: Time horizon in quarters