Unsmoothing Real Estate Returns: A Regime-switching Approach

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Bank of Thailand
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Reported Real Estate Return

- Appraisal-based return as opposed to market-traded
- *Smoothing* – Temporal aggregation and lagging effects
  - Serial correlation
  - Dampened volatility
- Real estate perceived as “safe” investment
- Implication in asset allocation / performance measurement
Unsmoothing Appraisal-based Return


- Fairly successful in that it raises volatility of (unsmoothed) real estate return

- Yet, real estate shown to have significantly better risk hedging characteristics than other asset classes (see e.g. Hudson-Wilson et al. 2003, Worzala & Sirmans 2003, Bond et al. 2007)

- Unsmoothed return still too smooth

- We found that this was only half the truth!
Our Regime-switching Approach

- Conventional method:
  - True return process
  - Smoothing equation
- Not completely satisfactory as it ignores non-linearity in performance data
- Our approach based on Threshold Autoregressive (TAR) model (Tong 1978, 1990)
  - Switching return: high volatility in “bad regime”, low in “good regime”
  - Switching smoothing: behavioral changes
- Results = direct and important practical implication
1. Base model
2. Regime-switching models
3. Estimation and Implementation
4. Results and Discussion
5. Conclusion
The Base Model

- **Measurement equation (Blundell & Ward 1987)**

\[ r_t^* = \alpha r_{t-1}^* + (1 - \alpha) r_t \]

where \( r_t^* \) = observed (smoothed) return, \( r_t \) the "true" return, and the smoothing coefficient \( \alpha \in (0, 1) \)

- **Given \( \alpha \), can calculate the unsmoothed return by**

\[ r_t = \frac{1}{1 - \alpha} \left( r_t^* - \alpha r_{t-1}^* \right) \]

- **Can also show that the “unsmoothed” variance is strictly increasing in \( \alpha \)**
The Base Model

Continued

- So far true return process irrelevant in a sense that $r_t$ may be obtained once $\alpha$ is known
- When $\alpha$ unknown, further information on $r_t$ required
- Practical approach: iid return, hence $\hat{\alpha} = \hat{\rho}_1$
- Return process (State equation)
  \[ r_t = \gamma + \phi r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2) \]
- Most studies assume $|\phi| < 1$
- Actual data probably not stationary
Non-linearity in Real Estate Return

Figure: Quarterly log-returns on IPD Index (Q4 1986 - Q4 2008)

Source: Investment Property Databank (IPD)
Regime-switching Approach

- **Smoothing equation**

\[ r_t^* = \alpha_t r_{t-1}^* + (1 - \alpha_t) r_t \]

- **State equation**

\[ r_t = \gamma_t + \phi_t r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2) \]

- **Regime-switching parameters**

\[ \alpha_t = \begin{cases} \alpha_1, & z_{1t-1} > c_1, \\ \alpha_2, & z_{1t-1} \leq c_1. \end{cases} \]

\[ (\gamma_t, \phi_t) = \begin{cases} (\gamma_1, \phi_1), & z_{2t-1} > c_2, \\ (\gamma_2, \phi_2), & z_{2t-1} \leq c_2. \end{cases} \]

where \( z_{it} \) = an exogenous observable regime indicator
Economic Identification

- Switching in the smoothing equation
  - Probably due to behavioural shifts of the appraisal agency
  - Different arrival rates of new information in “good” and “bad” regimes
  - Our model more generalised than that of Chaplin (1997)
- Switching return
  - Tied to changes in the macroeconomic environment
  - Time-varying volatility
- Open to a large number of plausible regime indicators
- Not necessarily the same regime indicator for smoothing and return
Potential Regime Indicators
Drivers of UK Real Estate Return

- Three-month LIBOR rate (end of period)
- GDP growth (nominal)
- SA employment
- Inflation – RPI excluding mortgage interest
- Log-return on FT All Share Total Return index
- Initial Yield index (rent to capital value)
- USD-GBP spot rate
Degree of Restrictions

- Conventional AR (AR-AR): $\alpha_t = \alpha, \gamma_t = \gamma, \phi_t = \phi$
- Switching return (AR-TAR): $\alpha_t = \alpha$
- Switching behaviour (TAR-AR): $\gamma_t = \gamma, \phi_t = \phi$
- Co-switching (TAR-TAR)
Estimation of AR-AR

- LS which iterates between the two equations (Cochrance-Orcutt-type)
- Implied AR(2) in reported return

\[ r_t^* = (1 - \alpha) \gamma + (\alpha + \phi) r_{t-1}^* - \alpha \phi r_{t-2}^* + \nu_t \]

with \( \nu_t = (1 - \alpha) \varepsilon_t \)

- Given \((\gamma, \phi)\), can estimate \(\alpha\) by LS
- Then use \(\hat{\alpha}\) to obtain \(r_t\)
- Use this unsmoothed return to estimate \((\gamma, \phi)\)
- Continue until coefficients converge, i.e. differ than previous value by less than 0.01
- Consistent (though not efficient)
Estimation of TAR-TAR

- Straightforward extension of the previous technique
- Implied TAR(2)

\[(1 - \phi_t L) (1 - \alpha_t L) r_t^* = (1 - \alpha_t) (\gamma_t + \varepsilon_t)\]

- Make use of an indicator function \(I_{it} = 1(z_{it} > c_i)\) such that

\[\alpha_t = \alpha_1 I_{1t-1} + \alpha_2 (1 - I_{1t-1})\]

\[(\gamma_t, \phi_t) = (\gamma_1, \phi_1) I_{2t-1} + (\gamma_2, \phi_2) (1 - I_{2t-1})\]

- Provides perfect discrimination between regimes
Estimation of TAR-TAR

Continued

- Initialisation: \((\gamma_0^1, \gamma_0^2, \phi_0^1, \phi_0^2, c_0^2)\)

- Built-in grid search to estimate the threshold level (not required when no switching involved)

- **Comment**: In practice actually easier to search over all possible values of \(z_{it}\) which are of \(O(T)\)

- \(c_i\) chosen such that the standard error of regression is minimised

- Consistency discussed in Franses & van Dijk (2000)

- Re-iterates till converged
## Estimation Results

### Table: AR-TAR (Switching Return)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\gamma_1$</th>
<th>$\phi_1$</th>
<th>$\gamma_2$</th>
<th>$\phi_2$</th>
<th>$c$</th>
<th>$\pi$</th>
<th>(Min,Max)</th>
<th>SSE</th>
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</thead>
<tbody>
<tr>
<td>TAR</td>
<td>0.51**</td>
<td>-1.25*</td>
<td>1.27**</td>
<td>2.38**</td>
<td>0.18</td>
<td>6.25</td>
<td>0.56</td>
<td>(2.83,15.25)</td>
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<td>LIBOR</td>
<td>0.07</td>
<td>(0.48)</td>
<td>(0.10)</td>
<td>(0.65)</td>
<td>(0.11)</td>
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<tr>
<td>INF</td>
<td>0.77**</td>
<td>0.25</td>
<td>-0.09</td>
<td>-0.22</td>
<td>0.95**</td>
<td>0.94</td>
<td>0.33</td>
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<td>(1.08)</td>
<td>(0.23)</td>
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<tr>
<td>FT</td>
<td>0.53**</td>
<td>2.22**</td>
<td>0.31**</td>
<td>-4.05**</td>
<td>1.77**</td>
<td>-1.54</td>
<td>0.76</td>
<td>(-32.0,18.84)</td>
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<td>(0.09)</td>
<td>(0.32)</td>
<td>(0.08)</td>
<td>(0.94)</td>
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<tr>
<td>GDP</td>
<td>0.81**</td>
<td>1.72</td>
<td>0.11</td>
<td>5.18</td>
<td>4.27*</td>
<td>-0.41</td>
<td>0.94</td>
<td>(-1.80,2.20)</td>
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<td>(0.12)</td>
<td>(0.99)</td>
<td>(0.08)</td>
<td>(6.66)</td>
<td>(1.86)</td>
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<tr>
<td>AR</td>
<td>0.94**</td>
<td>-1.35</td>
<td>0.12</td>
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<td>309.53</td>
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<tr>
<td></td>
<td>(0.04)</td>
<td>(2.82)</td>
<td>(0.15)</td>
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</tr>
</tbody>
</table>

Notes: (i) Newey-West HAC s.d. in parenthesis; (ii) * sig. at 5%, ** at 1%.
TAR on FT Returns
Implication on Real Estate Return

Figure: Quarterly log-returns on FT Index (Q4 1986 - Q4 2008)

“Good regime” positive mean, stationary

“Bad regime” negative mean, explosive
Quality of Regime Indicators
The 1990s & the recent crises

Figure: End-of-quarter LIBOR and Quarterly GDP Growth

“High state”

“Low state”

LIBOR

GDP Growth
AR-TAR Results

- Arguably, much more economically sound than AR-AR
- AR-AR: Smoothing = 0.94, i.e. the unsmoothed return will be highly volatile (at all times)
- AR-TAR: “Abnormal return” sieved from “normal return” where volatility is relatively low
- Usually, explosive return in one regime (bad regime), yet steady-state variance exists
- Knight & Satchell (2011): $\phi_1^2 > 1$ and $\phi_1^2 \pi + (1 - \pi) \phi_2^2 < 1$
Smoothed and Unsmoothed Returns

AR-AR vs AR-TAR

<table>
<thead>
<tr>
<th></th>
<th>$r^*$</th>
<th>$r_{AR}$</th>
<th>$r_{TAR}$</th>
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<tbody>
<tr>
<td>Mean</td>
<td>2.14</td>
<td>-1.15</td>
<td>1.90</td>
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<tr>
<td>Median</td>
<td>2.56</td>
<td>1.86</td>
<td>2.26</td>
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<tr>
<td>Maximum</td>
<td>8.03</td>
<td>89.19</td>
<td>13.07</td>
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<td>Minimum</td>
<td>-14.51</td>
<td>-177.01</td>
<td>-26.48</td>
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<td>Std. Dev.</td>
<td>3.27</td>
<td>34.11</td>
<td>4.99</td>
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<tr>
<td>Skewness</td>
<td>-1.96</td>
<td>-2.12</td>
<td>-2.82</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>10.54</td>
<td>13.16</td>
<td>16.61</td>
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</tbody>
</table>

Remark: Regime indicator = FT return
### Table: Estimated TAR-AR model

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha_1$</th>
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<th>$\gamma$</th>
<th>$\phi$</th>
<th>$c$</th>
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<td><strong>TAR</strong></td>
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<td></td>
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<td></td>
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<tr>
<td>LIBOR</td>
<td>1.22**</td>
<td>0.75**</td>
<td>3.69</td>
<td>-0.04</td>
<td>12.30</td>
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<td>(2.83,15.25)</td>
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<td>(0.20)</td>
<td>(2.71)</td>
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<td>(0.11)</td>
<td>(1.97)</td>
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<tr>
<td>FT</td>
<td>0.21**</td>
<td>1.97**</td>
<td>1.17</td>
<td>0.60**</td>
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<td>0.76</td>
<td>(-32.00,18.84)</td>
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<td>(0.08)</td>
<td>(0.25)</td>
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<tr>
<td>GDP</td>
<td>0.82**</td>
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<tr>
<td><strong>AR</strong></td>
<td>0.94**</td>
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Notes: (i) Newey-West HAC s.d. in parenthesis; (ii) * sig. at 5%, ** at 1%.
Switching Smoothing

Continued

- Basically,

  \[ \hat{\pi}\hat{\alpha}_1 + (1 - \pi)\hat{\alpha}_2 = \hat{\alpha} \]

- Excessive smoothing in “bad regime” i.e. high LIBOR / low FT returns / low GDP growth

- Psychological effect (?)

- But fits data slightly worse than AR-TAR (switching return)

- In some cases, lower (unconditional) volatility than smoothed return e.g. for FT return, s.d. according to TAR-AR = 2.83 less than 3.27 = s.d. of smoothed return
## Co-switching Model

**Table: Estimated TAR-TAR model**

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<tr>
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<th>$\gamma_2$</th>
<th>$\phi_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>SSE</th>
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<tbody>
<tr>
<td>TAR-TAR</td>
<td>1.42**</td>
<td>0.73**</td>
<td>-0.37</td>
<td>0.79**</td>
<td>3.05**</td>
<td>-0.04</td>
<td>6.21</td>
<td>11.31</td>
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<tr>
<td>LIBOR-LIBOR</td>
<td>(0.28)</td>
<td>(0.07)</td>
<td>(1.33)</td>
<td>(0.14)</td>
<td>(0.68)</td>
<td>(0.15)</td>
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<tr>
<td>FT-FT</td>
<td>0.72**</td>
<td>0.96**</td>
<td>3.36**</td>
<td>0.01</td>
<td>-7.13*</td>
<td>1.40**</td>
<td>-13.33</td>
<td>-1.20</td>
<td>183.16</td>
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<td></td>
<td>(0.07)</td>
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<td>(0.41)</td>
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<tr>
<td>LIBOR-FT</td>
<td>1.40**</td>
<td>0.56**</td>
<td>1.63**</td>
<td>0.35**</td>
<td>5.28**</td>
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<td>211.80</td>
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<td>(0.10)</td>
<td>(0.80)</td>
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Notes: (i) Newey-West HAC s.d. in parenthesis; (ii) * sig. at 5%, ** at 1%.
Unsmoothed Returns under Co-switching
AR-AR vs TAR-TAR

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<tr>
<td>Kurtosis</td>
<td>10.54</td>
<td>13.16</td>
<td>12.66</td>
</tr>
</tbody>
</table>

Remark: FT return = indicator for both smoothing and return equations
How does Co-switching work?

![Graph showing FT Returns and smoothing switching points](image)

- **True return switching point**
- **Smoothing switching point**
Comments on TAR-TAR

- Least restricted in this paper
- Explosive regime-switching behaviour either in the smoothing equation or in the return process, BUT not simultaneously
- From a theoretical viewpoint: Particular structure on the state probability when a single indicator governs both equations
- Supported by Hansen (1996, 1997) test results, especially when involved with FT returns
- Results driven by different combinations of the exogenous variables used, thereby opening up a new area of research
Implication on Asset Allocation

- The TAR-TAR unconditional variance still close to that of the conventional smoothing model
- However, **Time-varying behaviour** (conditional smoothing) masked by the latter
- More informative and also crucial to successful dynamic (active) asset allocation
- Also, sheds more light on the nature of real estate risk
- The impact in the extreme regimes, although probably short lived, profound to asset values
- After all, quality of risk measures dependent of accuracy of estimated smoothing coefficient(s)
Conclusions

- New unsmoothing technique for returns on an appraisal-based valuation index
- Clear evidence of regime effects and time-varying behavior in the commercial real estate returns
- Most promising results from the use of FT equity returns, LIBOR, and to a lesser extent, GDP growth
- TAR-TAR better than AR-AR according to SSE criteria (about 40% reduction)
- Applicable to other “illiquid” asset classes, e.g. hedge fund, venture capital, or even fine art!
Thank you!